## **Robot Mapping**

## FastSLAM – Feature-Based SLAM with Particle Filters

**Cyrill Stachniss** 



#### **Particle Filter**

- Non-parametric recursive Bayes filter
- Posterior is represented by a set of weighted samples
- Can model arbitrary distributions
- Works well in low-dimensional spaces
- 3-Step procedure
  - Sampling from proposal
  - Importance Weighting
  - Resampling

## **Particle Filter Algorithm**

1. Sample the particles from the proposal distribution

$$x_t^{[j]} \sim \pi(x_t \mid \ldots)$$

2. Compute the importance weights

$$w_t^{[j]} = \frac{target(x_t^{[j]})}{proposal(x_t^{[j]})}$$

3. Resampling: Draw sample i with probability  $w_t^{[i]}$  and repeat J times

## **Particle Representation**

A set of weighted samples

$$\mathcal{X} = \left\{ \left\langle x^{[i]}, w^{[i]} \right\rangle \right\}_{i=1,\dots,N}$$

- Think of a sample as one hypothesis about the state
- For feature-based SLAM:

$$x = (x_{1:t}, m_{1,x}, m_{1,y}, \dots, m_{M,x}, m_{M,y})^{T}$$
poses landmarks

## **Dimensionality Problem**

Particle filters are effective in low dimensional spaces as the likely regions of the state space need to be covered with samples.

$$x = (x_{1:t}, m_{1,x}, m_{1,y}, \dots, m_{M,x}, m_{M,y})^T$$
  
high-dimensional

# Can We Exploit Dependencies Between the Different Dimensions of the State Space?

$$x_{1:t}, m_1, \ldots, m_M$$

## If We Know the Poses of the Robot, Mapping is Easy!

$$x_{1:t}, m_1, \ldots, m_M$$

## **Key Idea**

$$x_{1:t}, m_1, \ldots, m_M$$

If we use the particle set only to model the robot's path, each sample is a path hypothesis. For each sample, we can compute an individual map of landmarks.

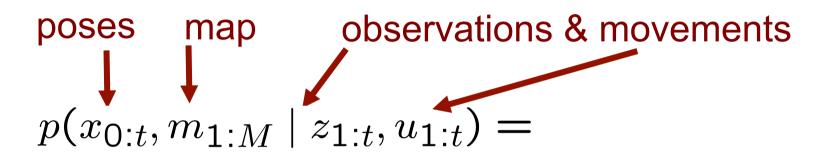
#### Rao-Blackwellization

Factorization to exploit dependencies between variables:

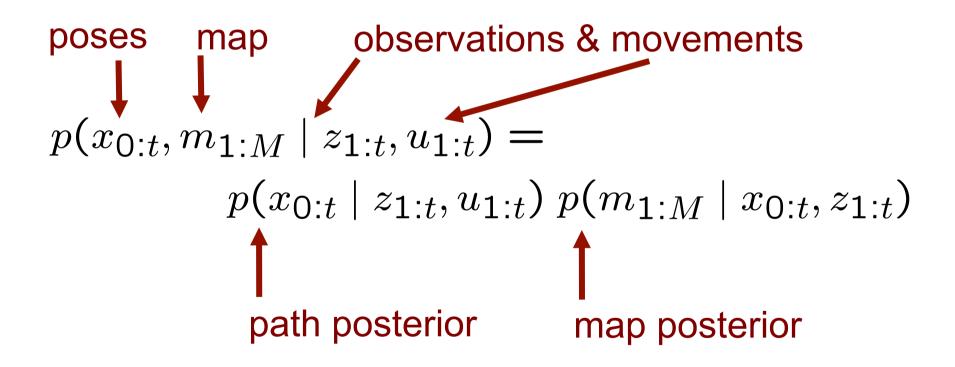
$$p(a,b) = p(b \mid a) p(a)$$

• If  $p(b \mid a)$  can be computed efficiently, represent only p(a) with samples and compute  $p(b \mid a)$  for every sample

Factorization of the SLAM posterior



Factorization of the SLAM posterior



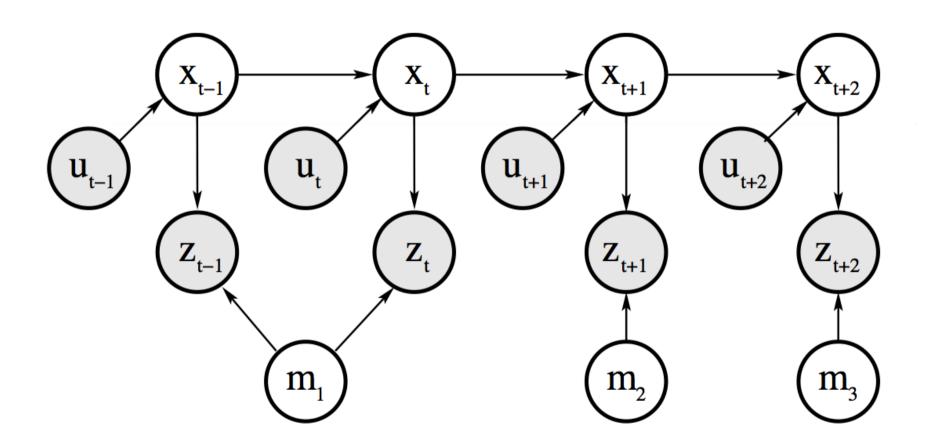
Factorization of the SLAM posterior

$$p(x_{0:t}, m_{1:M} \mid z_{1:t}, u_{1:t}) =$$

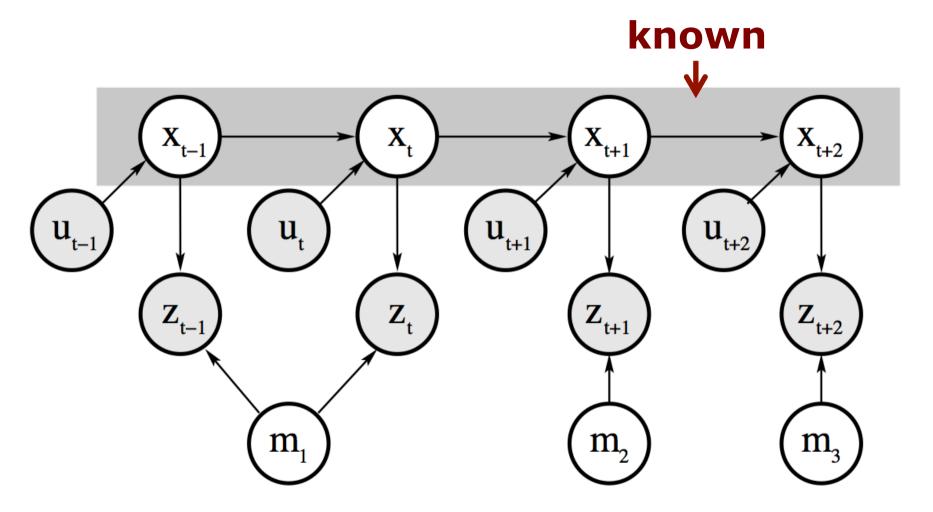
$$p(x_{0:t} \mid z_{1:t}, u_{1:t}) \ p(m_{1:M} \mid x_{0:t}, z_{1:t})$$

How to compute this term efficiently?

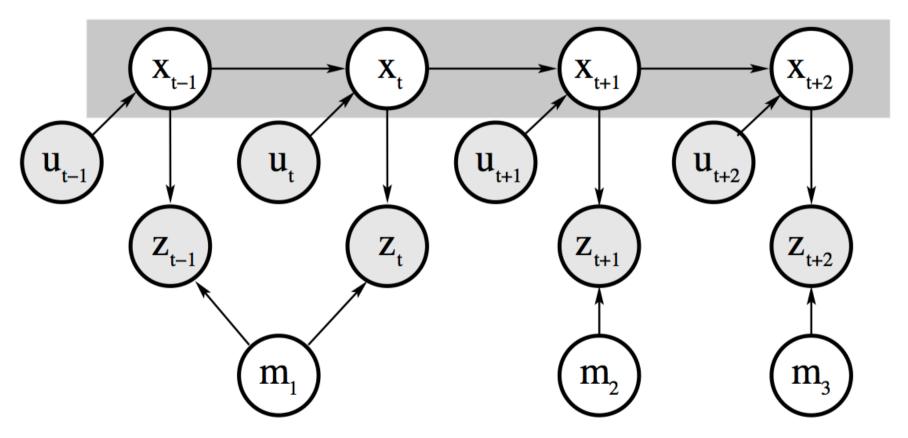
## **Revisit the Graphical Model**



## **Revisit the Graphical Model**



## Landmarks are Conditionally Independent Given the Poses



Landmark variables are all disconnected (i.e. independent) given the robot's path

Factorization of the SLAM posterior

$$p(x_{0:t}, m_{1:M} \mid z_{1:t}, u_{1:t}) =$$

$$p(x_{0:t} \mid z_{1:t}, u_{1:t}) \ p(m_{1:M} \mid x_{0:t}, z_{1:t})$$



Landmarks are conditionally independent given the poses

Factorization of the SLAM posterior

$$p(x_{0:t}, m_{1:M} \mid z_{1:t}, u_{1:t}) =$$

$$p(x_{0:t} \mid z_{1:t}, u_{1:t}) p(m_{1:M} \mid x_{0:t}, z_{1:t})$$

$$p(x_{0:t} \mid z_{1:t}, u_{1:t}) \prod_{i=1}^{M} p(m_i \mid x_{0:t}, z_{1:t})$$

Factorization of the SLAM posterior

$$p(x_{0:t}, m_{1:M} \mid z_{1:t}, u_{1:t}) =$$

$$p(x_{0:t} \mid z_{1:t}, u_{1:t}) p(m_{1:M} \mid x_{0:t}, z_{1:t})$$

$$p(x_{0:t} \mid z_{1:t}, u_{1:t}) \prod_{i=1}^{M} p(m_i \mid x_{0:t}, z_{1:t})$$

#### 2-dimensional EKFs!

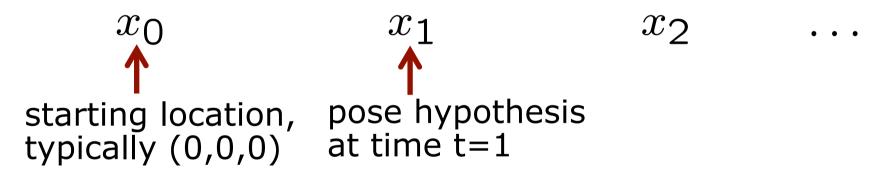
Factorization of the SLAM posterior

$$p(x_{0:t}, m_{1:M} \mid z_{1:t}, u_{1:t}) = \\ p(x_{0:t} \mid z_{1:t}, u_{1:t}) \ p(m_{1:M} \mid x_{0:t}, z_{1:t}) \\ \frac{p(x_{0:t} \mid z_{1:t}, u_{1:t})}{\int} \prod_{i=1}^{M} p(m_i \mid x_{0:t}, z_{1:t}) \\ \text{particle filter similar to MCL}$$

#### 2-dimensional EKFs!

## Modeling the Robot's Path

- Sample-based representation for  $p(x_{0:t} \mid z_{1:t}, u_{1:t})$
- Each sample is a path hypothesis



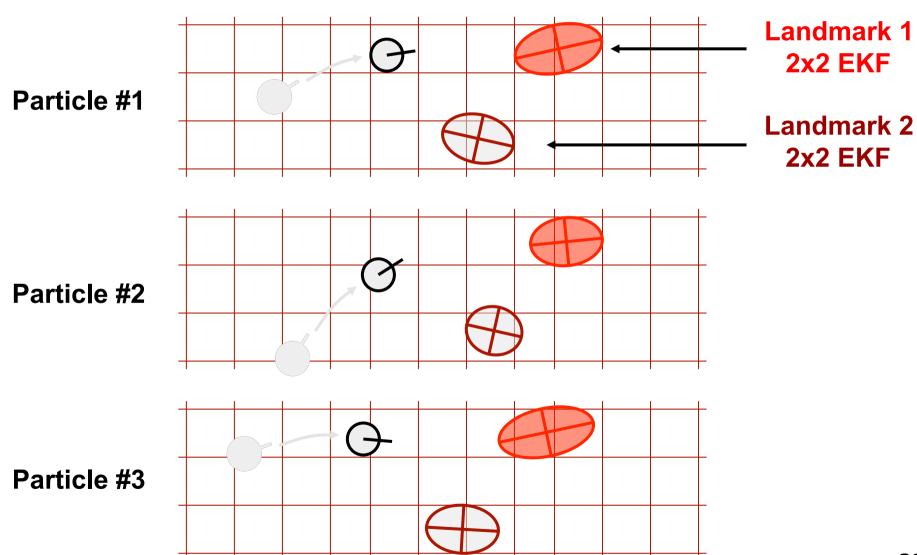
- Past poses of a sample are not revised
- No need to maintain past poses in the sample set

#### **FastSLAM**

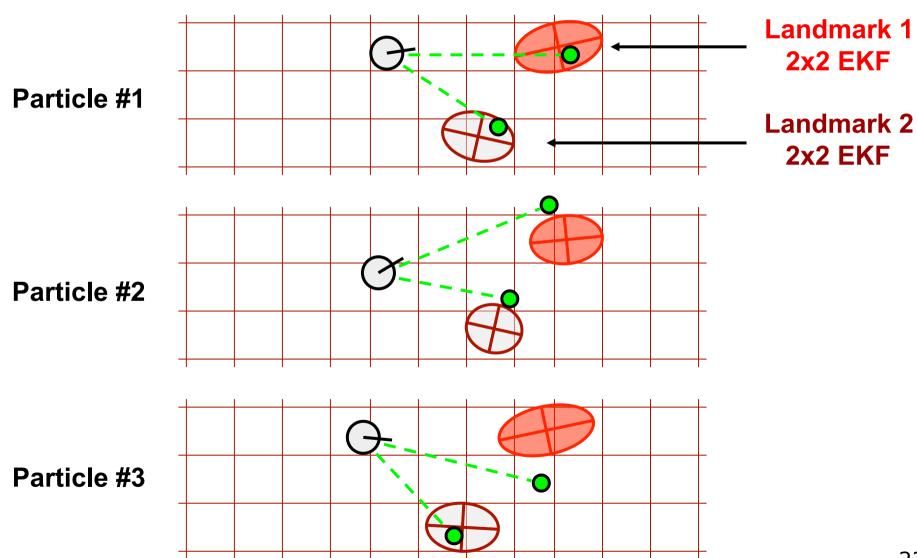
- Proposed by Montemerlo et al. in 2002
- Each landmark is represented by a 2x2 EKF
- Each particle therefore has to maintain M individual EKFs



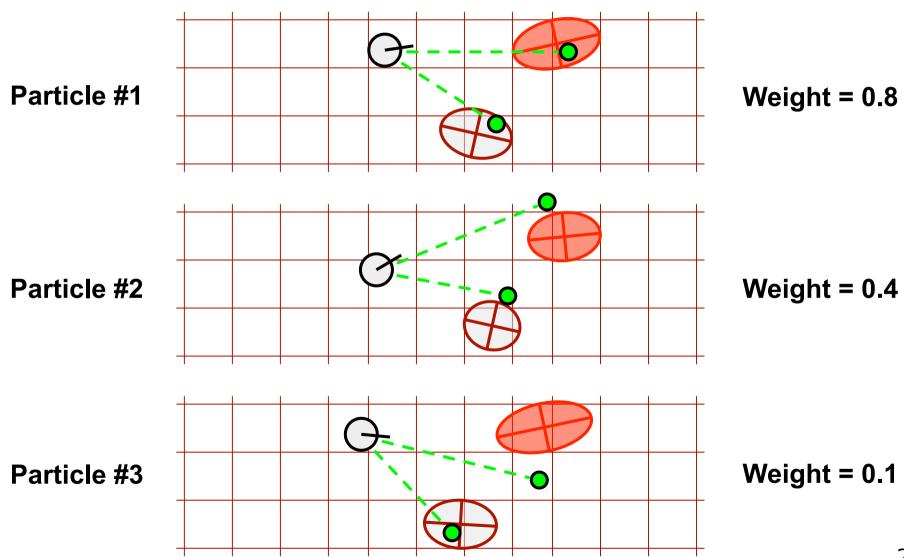
## FastSLAM - Action Update



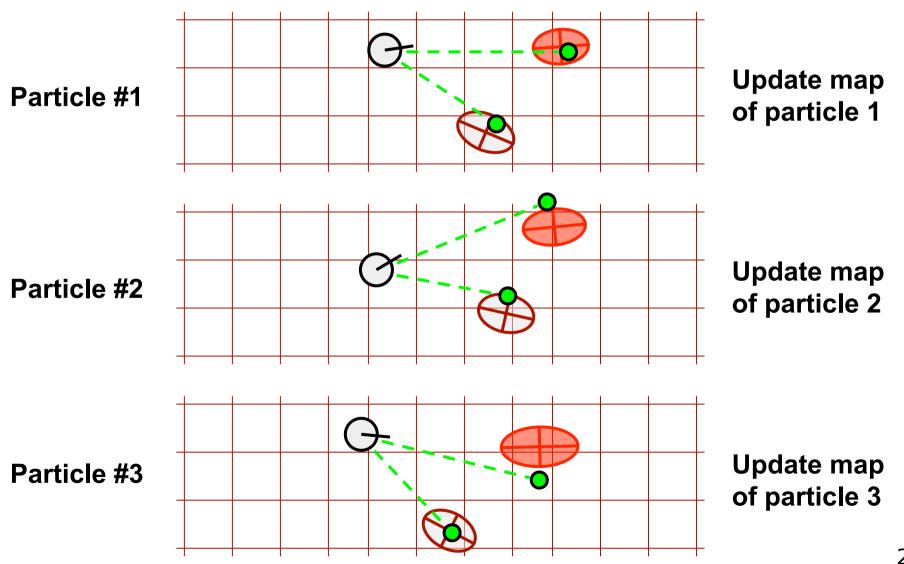
## FastSLAM – Sensor Update



## FastSLAM - Sensor Update



### FastSLAM - Sensor Update



### **Key Steps of FastSLAM 1.0**

 Extend the path posterior by sampling a new pose for each sample

$$x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$$

Compute particle weight

exp. observation

$$w^{[k]} = |2\pi Q|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T Q^{-1} (z_t - \hat{z}^{[k]})\right\}$$

measurement covariance

- Update belief of observed landmarks (EKF update rule)
- Resample

#### FastSLAM 1.0 - Part 1

```
1: FastSLAM1.0_known_correspondence(z_t, c_t, u_t, \mathcal{X}_{t-1}):

2: for k = 1 to N do // loop over all particles

3: Let \left\langle x_{t-1}^{[k]}, \left\langle \mu_{1,t-1}^{[k]}, \Sigma_{1,t-1}^{[k]} \right\rangle, \ldots \right\rangle be particle k in \mathcal{X}_{t-1}

4: x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t) // sample pose
```

#### FastSLAM 1.0 - Part 1

```
FastSLAM1.0_known_correspondence(z_t, c_t, u_t, \mathcal{X}_{t-1}):
                   k = 1 \text{ to } N \text{ do} // loop over all particles Let \left\langle x_{t-1}^{[k]}, \left\langle \mu_{1,t-1}^{[k]}, \Sigma_{1,t-1}^{[k]} \right\rangle, \ldots \right\rangle be particle k \text{ in } \mathcal{X}_{t-1}
              for k = 1 to N do
                   x_{t}^{[k]} \sim p(x_{t} \mid x_{t-1}^{[k]}, u_{t})
4:
                                                             // sample pose
                                                                           // observed feature
                   j=c_t
                   if feature j never seen before
                       \mu_{i,t}^{[k]} = h^{-1}(z_t, x_t^{[k]})
                                                       // initialize mean
// calculate Jacobian
                       H = h'(\mu_{i,t}^{[k]}, x_t^{[k]})
8:
                       \Sigma_{j,t}^{[k]} = H^{-1} Q_t (H^{-1})^T // initialize covariance w^{[k]} = p_0 // default importance
9:
                                                                           // default importance weight
10:
11:
                    else
```

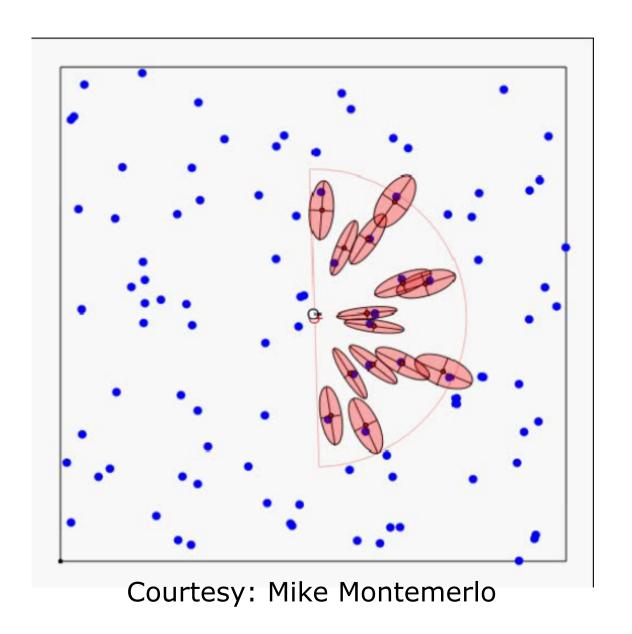
#### FastSLAM 1.0 - Part 2

```
11:
                      else
                          \langle \mu_{i,t}^{[k]}, \Sigma_{i,t}^{[k]} \rangle = EKF\text{-}Update(\dots) // update landmark
12:
                         w^{[k]} = |2\pi Q|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T Q^{-1} (z_t - \hat{z}^{[k]})\right\}
13:
  measurement cov. Q = H \sum_{i,t-1}^{[k]} H^T + Q_t exp. observation
14:
                      endif
15:
                      for all unobserved features j' do
                         \langle \mu_{i',t}^{[k]}, \Sigma_{i',t}^{[k]} \rangle = \langle \mu_{i',t-1}^{[k]}, \Sigma_{i',t-1}^{[k]} \rangle // leave unchanged
16:
17:
                      endfor
18:
               endfor
          \mathcal{X}_t = \text{resample}\left(\left\langle x_t^{[k]}, \left\langle \mu_{1,t}^{[k]}, \Sigma_{1,t}^{[k]} \right\rangle, \dots, w^{[k]} \right\rangle_{k=1,\dots,N}\right)
19:
20:
               return \mathcal{X}_t
```

## FastSLAM 1.0 - Part 2 (long)

```
11:
                    else
                       w^{[k]} = |2\pi Q|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T\right\}
                                                  Q^{-1}(z_t - \hat{z}^{[k]}) // importance factor
  19:
                    endif
                    for all unobserved features j' do
  20:
                        \langle \mu_{i',t}^{[k]}, \Sigma_{i',t}^{[k]} \rangle = \langle \mu_{i',t-1}^{[k]}, \Sigma_{i',t-1}^{[k]} \rangle // leave unchanged
  21:
  23:
                    endfor
               endfor
  24:
            \mathcal{X}_t = \text{resample}\left(\left\langle x_t^{[k]}, \left\langle \mu_{1,t}^{[k]}, \Sigma_{1,t}^{[k]} \right\rangle, \dots, w^{[k]} \right\rangle_{k=1,\dots,N}\right)
  25:
  26:
               return \mathcal{X}_t
```

### **FastSLAM** in Action



## The Weight is a Result From the Importance Sampling Principle

- Importance weight is given by the ratio of target and proposal in  $\boldsymbol{x}^{[k]}$
- See: importance sampling principle

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

The target distribution is

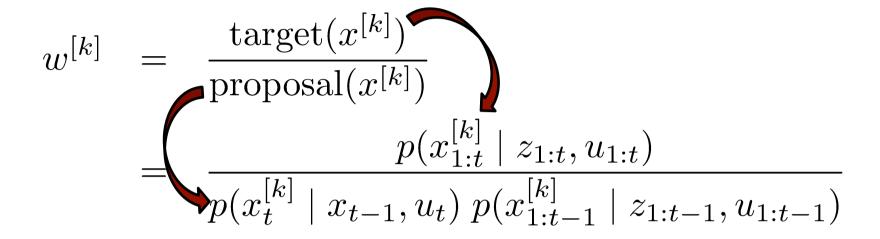
$$p(x_{1:t} \mid z_{1:t}, u_{1:t})$$

The proposal distribution is

$$p(x_{1:t} \mid z_{1:t-1}, u_{1:t})$$

Proposal is used step-by-step

$$p(x_{1:t} \mid z_{1:t-1}, u_{1:t}) = \underbrace{p(x_t \mid x_{t-1}, u_t)}_{\text{from } \mathcal{X}_{t-1} \text{ to } \bar{\mathcal{X}}_t} \underbrace{p(x_{1:t-1} \mid z_{1:t-1}, u_{1:t-1})}_{\mathcal{X}_{t-1}}$$



$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

$$= \frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t})}{p(x_t^{[k]} \mid x_{t-1}, u_t) p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

**Bayes rule + factorization** 

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

$$= \frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t})}{p(x_t^{[k]} \mid x_{t-1}, u_t) p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$= \frac{\eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}) p(x_t \mid x_{t-1}^{[k]}, u_t)}{p(x_t^{[k]} \mid x_{t-1}^{[k]}, u_t)}$$

$$= \frac{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

$$= \frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t})}{p(x_t^{[k]} \mid x_{t-1}, u_t) p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$= \frac{\eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}) p(x_t \mid x_{t-1}^{[k]}, u_t)}{p(x_t^{[k]} \mid x_{t-1}^{[k]}, u_t)}$$

$$= \frac{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

$$= \frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t})}{p(x_t^{[k]} \mid x_{t-1}, u_t) p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$= \frac{\eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}) p(x_t \mid x_{t-1}^{[k]}, u_t)}{p(x_t^{[k]} \mid x_{t-1}^{[k]}, u_t)}$$

$$= \frac{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}{p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$= \eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1})$$

 Integrating over the pose of the observed landmark leads to

$$w^{[k]} = \eta \ p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1})$$

$$= \eta \int p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}, m_j) \ p(m_j \mid x_{1:t}^{[k]}, z_{1:t-1}) \ dm_j$$

 Integrating over the pose of the observed landmark leads to

$$w^{[k]}$$

$$= \eta \ p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1})$$

$$= \eta \int p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}, m_j) \ p(m_j \mid x_{1:t}^{[k]}, z_{1:t-1}) \ dm_j$$

$$= \eta \int p(z_t \mid x_t^{[k]}, m_j) \ p(m_j \mid x_{1:t-1}^{[k]}, z_{1:t-1}) \ dm_j$$

 Integrating over the pose of the observed landmark leads to

$$w^{[k]} = \eta \ p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1})$$

$$= \eta \int p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}, m_j) \ p(m_j \mid x_{1:t}^{[k]}, z_{1:t-1}) \ dm_j$$

$$= \eta \int \underbrace{p(z_t \mid x_t^{[k]}, m_j)}_{\mathcal{N}(z_t; \hat{z}^{[k]}, Q_t)} \underbrace{p(m_j \mid x_{1:t-1}^{[k]}, z_{1:t-1})}_{\mathcal{N}(m_j; \mu_{j,t-1}^{[k]}, \Sigma_{j,t-1}^{[k]})} dm_j$$

This leads to

$$w^{[k]} = \eta \int \underbrace{p(m_j \mid x_{1:t-1}^{[k]}, z_{1:t-1})}_{\mathcal{N}(m_j; \mu_{j,t-1}^{[k]}, \Sigma_{j,t-1}^{[k]})} \underbrace{p(z_t \mid x_t^{[k]}, m_j)}_{\mathcal{N}(z_t; \hat{z}^{[k]}, Q_t)} dm_j$$

$$Q = H \sum_{j,t-1}^{[k]} H^T + Q_t$$

measurement covariance (pose uncertainty of the landmark estimate plus measurement noise)

This leads to

$$w^{[k]} = \eta \int \underbrace{p(m_j \mid x_{1:t-1}^{[k]}, z_{1:t-1})}_{\mathcal{N}(m_j; \mu_{j,t-1}^{[k]}, \Sigma_{j,t-1}^{[k]})} \underbrace{p(z_t \mid x_t^{[k]}, m_j)}_{\mathcal{N}(z_t; \hat{z}^{[k]}, Q_t)} dm_j$$

$$Q = H \sum_{j,t-1}^{[k]} H^T + Q_t$$

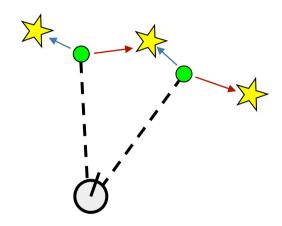
$$w^{[k]} \simeq |2\pi Q|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T Q^{-1} (z_t - \hat{z}^{[k]})\right\}$$

#### FastSLAM 1.0 - Part 2

```
11:
                        else
                             \langle \mu_{i,t}^{[k]}, \Sigma_{i,t}^{[k]} \rangle = EKF\text{-}Update(\dots) // update landmark
12:
                          w^{[k]} = |2\pi Q|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T Q^{-1} (z_t - \hat{z}^{[k]})\right\}
13:
14:
                        endif
15:
                        for all unobserved features j' do
                            \langle \mu_{i',t}^{[k]}, \Sigma_{i',t}^{[k]} \rangle = \langle \mu_{i',t-1}^{[k]}, \Sigma_{i',t-1}^{[k]} \rangle // leave unchanged
16:
17:
                        endfor
18:
                 endfor
                \mathcal{X}_t = \text{resample}\left(\left\langle x_t^{[k]}, \left\langle \mu_{1,t}^{[k]}, \Sigma_{1,t}^{[k]} \right\rangle, \dots, w^{[k]} \right\rangle_{k=1,\dots,N}\right)
19:
20:
                 return \mathcal{X}_t
```

#### **Data Association Problem**

Which observation belongs to which landmark?



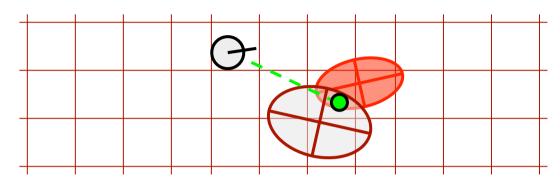
- More than one possible association
- Potential data associations depend on the pose of the robot

## Particles Support for Multi-Hypotheses Data Association

Decisions on a perparticle basis



#### Per-Particle Data Association

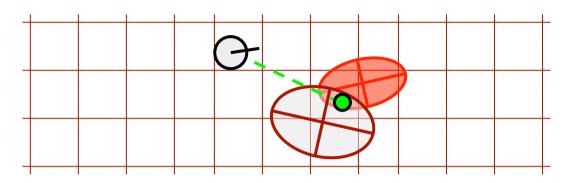


Was the observation generated by the **red** or by the **brown** landmark?

P(observation|red) = 0.3

P(observation|brown) = 0.7

#### Per-Particle Data Association



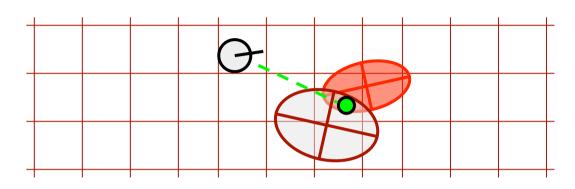
Was the observation generated by the **red** or by the **brown** landmark?

P(observation|red) = 0.3

P(observation|brown) = 0.7

- Two options for per-particle data association
  - Pick the most probable match
  - Pick an random association weighted by the observation likelihoods
- If the probability for an assignment is too low, generate a new landmark

#### Per-Particle Data Association



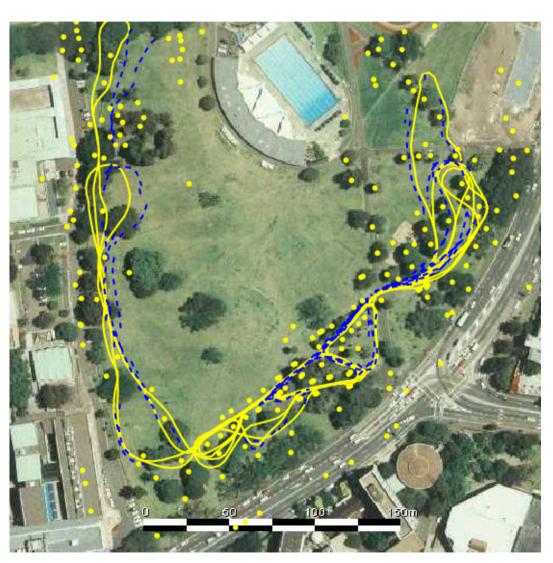
Was the observation generated by the **red** or by the **brown** landmark?

- Multi-modal belief
- Pose error is factored out of data association decisions
- Simple but effective data association
- Big advantage of FastSLAM over EKF

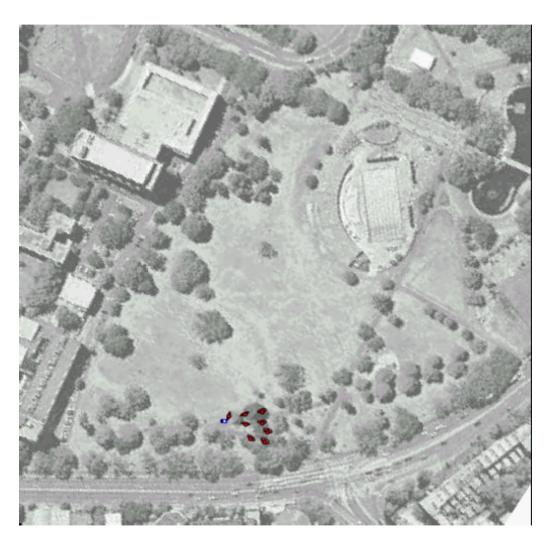
#### **Results – Victoria Park**

- 4 km traverse
- < 2.5 m RMS position error</p>
- 100 particles

Blue = GPS Yellow = FastSLAM

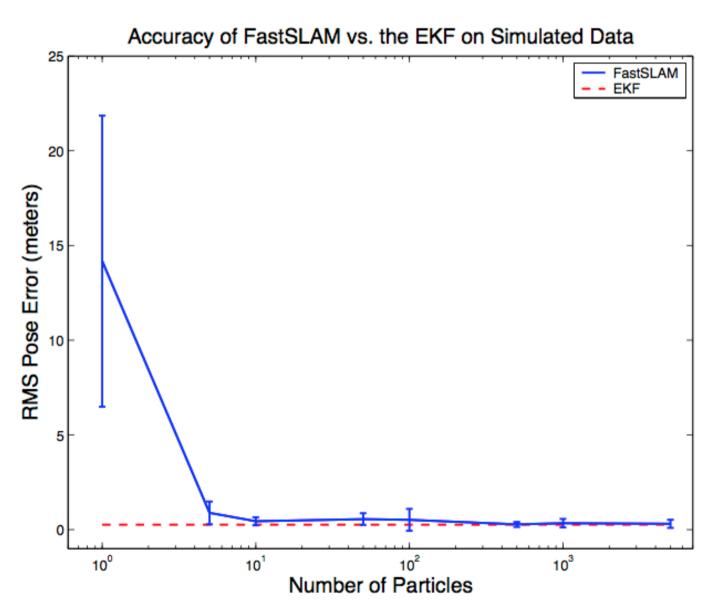


## Results - Victoria Park (Video)

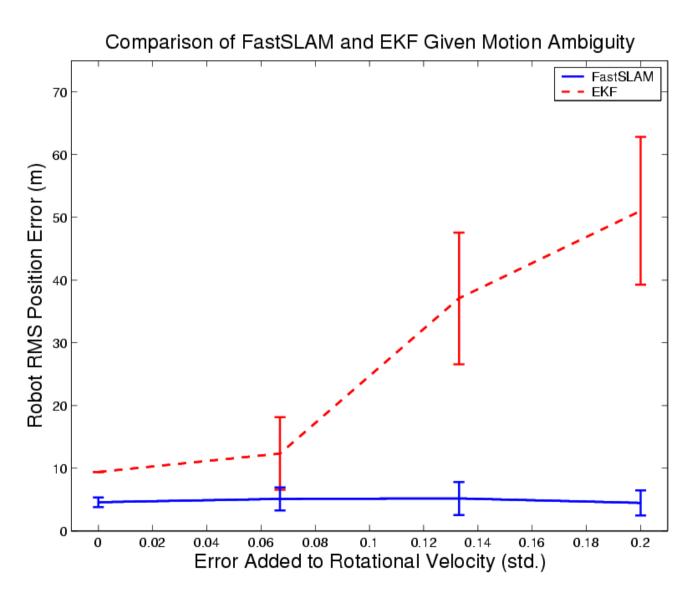


Courtesy: Mike Montemerlo

## Results (Sample Size)



## **Results (Motion Uncertainty)**



#### FastSLAM 1.0 Summary

- Use a particle filter to model the belief
- Factors the SLAM posterior into lowdimensional estimation problems
- Model only the robot's path by sampling
- Compute the landmarks given the path
- Per-particle data association
- No robot pose uncertainty in the perparticle data association

# FastSLAM Complexity – Simple Implementation

 Update robot particles based on the control

$$\mathcal{O}(N)$$

Incorporate an observation into the Kalman filters

$$\mathcal{O}(N)$$

Resample particle set

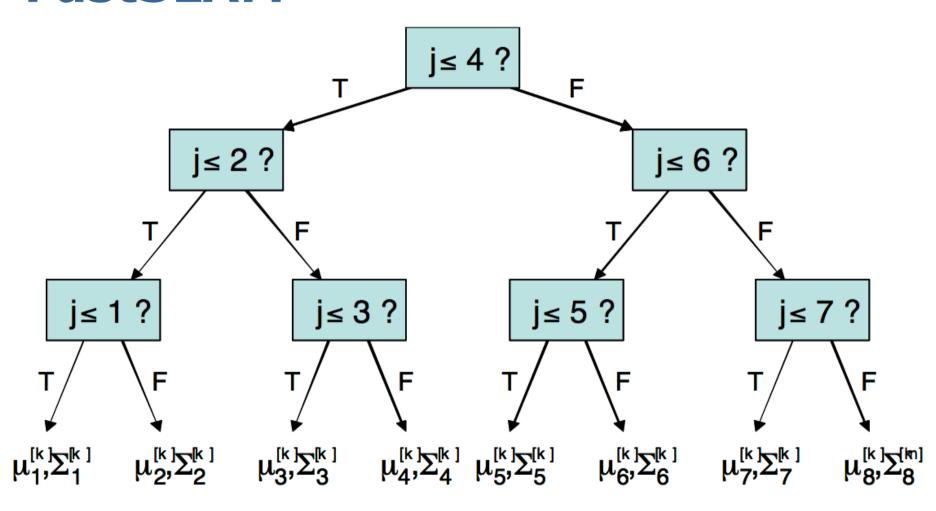
$$\mathcal{O}(NM)$$

N = Number of particles

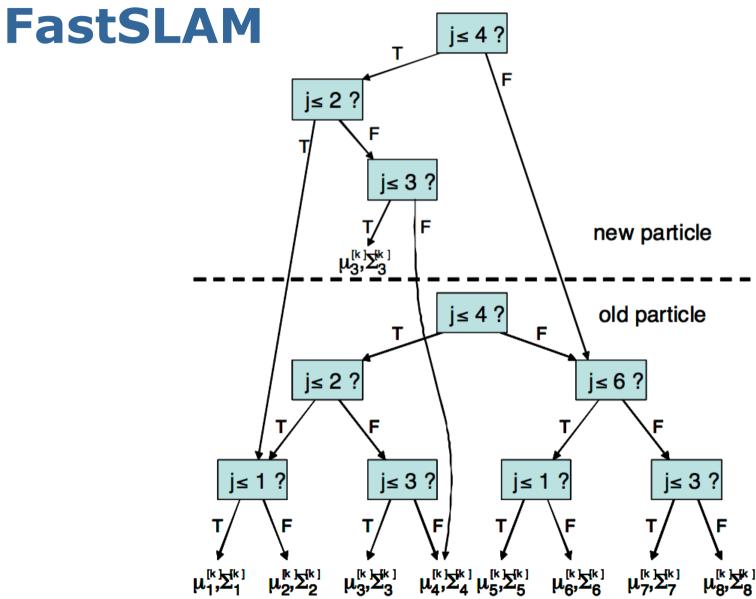
**M** = Number of map features

$$\mathcal{O}(NM)$$

## A Better Data Structure for FastSLAM



A Better Data Structure for



## **FastSLAM Complexity**

Update robot particles based on the control

 $\mathcal{O}(N)$ 

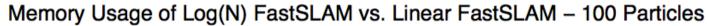
• Incorporate an observation  $\mathcal{O}(N\log M)$ into the Kalman filters

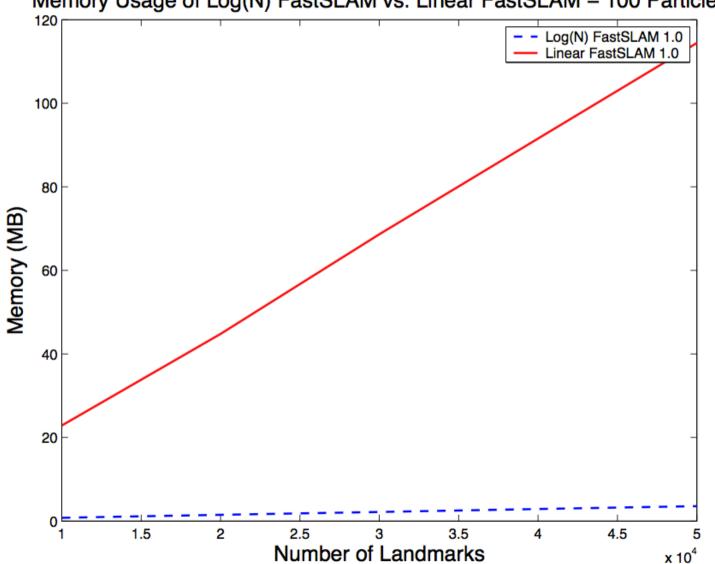
Resample particle set

 $\mathcal{O}(N \log M)$ 

**N** = Number of particles **M** = Number of map features  $\mathcal{O}(N \log M)$ 

## **Memory Complexity**





#### FastSLAM 1.0

 FastSLAM 1.0 uses the motion model as the proposal distribution

$$x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$$

Is there a better distribution to sample from?

#### FastSLAM 1.0 to FastSLAM 2.0

FastSLAM 1.0 uses the motion model as the proposal distribution

$$x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$$

- FastSLAM 2.0 considers also the measurements during sampling
- Especially useful if an accurate sensor is used (compared to the motion noise)

## FastSLAM 2.0 (Informally)

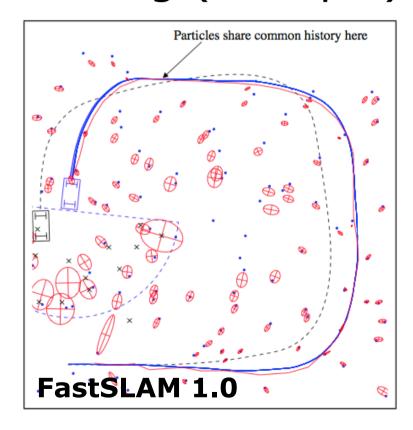
FastSLAM 2.0 samples from

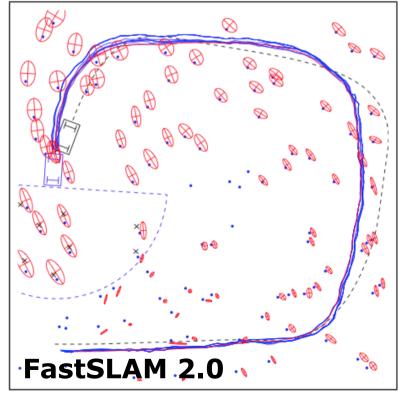
$$x_t^{[k]} \sim p(x_t \mid x_{1:t-1}^{[k]}, u_{1:t}, z_{1:t})$$

- Results in a more peaked proposal distribution
- Less particles are required
- More robust and accurate
- But more complex...

#### **FastSLAM Problems**

- How to determine the sample size?
- Particle deprivation, especially when closing (multiple) loops





#### **FastSLAM Summary**

- Particle filter-based SLAM
- Rao-Blackwellization: model the robot's path by sampling and compute the landmarks given the poses
- Allow for per-particle data association
- FastSLAM 1.0 and 2.0 differ in the proposal distribution
- Complexity  $\mathcal{O}(N \log M)$

#### **FastSLAM Results**

- Scales well (1 million+ features)
- Robust to ambiguities in the data association
- Advantages compared to the classical EKF approach (especially with nonlinearities)

#### Literature

#### **FastSLAM**

- Thrun et al.: "Probabilistic Robotics",
   Chapter 13.1-13.3 + 13.8 (see errata!)
- Montemerlo, Thrun, Kollar, Wegbreit: FastSLAM: A Factored Solution to the Simultaneous Localization and Mapping Problem, 2002
- Montemerlo and Thrun: Simultaneous Localization and Mapping with Unknown Data Association Using FastSLAM, 2003