

$$\begin{array}{c}
 \overset{0}{\parallel} \\
 x_1 \longrightarrow x_2 \longrightarrow x_3 \\
 \overset{1}{\parallel} \qquad \qquad \qquad \overset{2}{\parallel} \\
 z_{12} = 1m \qquad z_{23} = 1m \\
 \Omega_{12} = 1 \qquad \Omega_{23} = 0.5
 \end{array}$$

LS SLAM example
 Cyrill Stachniss
 Robotics II WS 11/12

initial guess $x_1 = x_2 = x_3 = 0$

$$e_{ij} = z_{ij} - (x_j - x_i)$$

$$e_{12} = z_{12} - (x_2 - x_1) = 1 - (0 - 0) = 1$$

$$e_{23} = z_{23} - (x_3 - x_2) = 1 - (0 - 0) = 1$$

$$J_{12} = \begin{pmatrix} \frac{\partial e_{12}}{\partial x_1} & \frac{\partial e_{12}}{\partial x_2} & \frac{\partial e_{12}}{\partial x_3} \end{pmatrix} = \begin{pmatrix} +1 & -1 & 0 \end{pmatrix}$$

$$J_{23} = \begin{pmatrix} 0 & +1 & -1 \end{pmatrix}$$

$$\begin{aligned}
 b^T &= \sum_i e_{ij}^T \Omega_{ij} J_{ij} = 1 \cdot 1 \cdot (1 \ -1 \ 0) + 1 \cdot \frac{1}{2} \cdot (0 \ +1 \ -1) \\
 &= \begin{pmatrix} 1 & -0.5 & -0.5 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 H &= \sum_i J_{ij}^T \Omega_{ij} J_{ij} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} 1 \cdot (1 \ -1 \ 0) + \begin{pmatrix} 0 \\ +1 \\ -1 \end{pmatrix} 0.5 \cdot (0 \ 1 \ -1) \\
 &= \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{pmatrix} \\
 &= \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1.5 & -0.5 \\ 0 & -0.5 & 0.5 \end{pmatrix}
 \end{aligned}$$

$$\Delta x = -H \setminus b \Rightarrow \text{error } \det(H) = 0$$

$$\Rightarrow H \leftarrow H + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \leftarrow \text{fix } x_1 = \text{current pose}$$

$$\Rightarrow \Delta x = -H \setminus b \Rightarrow \Delta x = (0, 1, 2) \Rightarrow \begin{array}{l} x_1 = 0 \\ x_2 = 1 \\ x_3 = 2 \end{array}$$