Theoretical Computer Science (Bridging Course)

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Exercise Sheet 0

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Exercise 0.1 (Proof by contradiction)

Prove the following statement by *contradiction*

> Let $q \in \mathbb{Q}$ and $x \in \mathbb{R} \setminus \mathbb{Q}$, then $q - x \in \mathbb{R} \setminus \mathbb{Q}$.

That is, the difference of any rational number and any irrational number is irrational.

Exercise 0.2 (Proofs by induction)

Prove *by induction* that the following statements hold for every $n \in \mathbb{N}^+$ (the set of positive integers).

- $\sum_{i=1}^{n} i^2 = \frac{n \cdot (n+1) \cdot (2n+1)}{6}$
- $1 - x^n = (1 - x)(1 + x + ... + x^{n-1})$.

Please make clear what is the *base case*, the *induction hypothesis* and the *induction step*.

Exercise 0.3 (Sets)

Let $E_1, ..., E_N$ be an arbitrary finite collection of sets. Show that

$$F \cup \left(\bigcap_{n=1}^{N} E_n \right) = \bigcap_{n=1}^{N} (F \cup E_n).$$