

## Theoretical Computer Science (Bridging Course)

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### Exercise Sheet 3

**Due: 20th November 2014**

**Exercise 3.1** (Regular languages, Pumping lemma)

Are the following languages regular? Prove it.

- (a)  $L := \{a^i b^j a^{ij} \mid i, j \geq 0\}$ .
- (b)  $L := \{b^2 a^n b^m a^3 \mid m, n \geq 0\}$ .
- (c)  $L := \{a^{k^3} \mid k \geq 0\}$ .

**Exercise 3.2** (Pumping Lemma)

Find the minimum pumping length of the languages  $L(\mathcal{R})$  where

- (a)  $\mathcal{R} = \mathcal{R}_1 := 0^*101^*$ .
- (b)  $\mathcal{R} = \mathcal{R}_2 := 10^*1$ .
- (c)  $\mathcal{R} = \mathcal{R}_1 \cup \mathcal{R}_2$ .

**Exercise 3.3** (Context-free languages)

- (a) Provide a context-free grammar  $G = (V, \Sigma, R, S)$  that generates the language of palindromes over an alphabet  $\Xi$ .
- (b) Prove that  $L(G) = L_{pal}$ .
- (c) Consider the context-free grammar  $(\{X, Y\}, \{0, 1\}, R, X)$  where  $R$  is defined as follows

$$\begin{aligned} X &\rightarrow \epsilon \mid 1, \\ X &\rightarrow 1 X 1 \mid Y, \\ Y &\rightarrow \epsilon \mid 0, \\ Y &\rightarrow 0 Y 0. \end{aligned}$$

Which language does this context-free grammar generate?