Theoretical Computer Science (Bridging Course)

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Winter semester 2014/2015

Exercise Sheet 5
Due: 4th December 2014

Exercise 5.1 (Context-free grammars, Pushdown automata)

- Construct a grammar that generates the regular expressions over an alphabet Ξ.

  Hint: You can think of a regular expression like a sequence of element in the alphabet $\Xi \cup \{\epsilon, \emptyset, \cup, \circ, ^*, (,), ^+\}$.

- (*Not mandatory*) Can you design an automation that checks whether a regular expression is well formed?

  Hint: Check slide 61 in 4.pdf.

Exercise 5.2 (Chomsky normal form)

Let $\mathcal{G} = (V, \Sigma, \mathcal{R}, S)$ be a context-free grammar, we call *sentential form* a word $\omega \in (\Sigma \cup V)^*$ so that ω is derived by applying a sequence of production rules in $\mathcal{R}$. Roughly speaking, sentential forms are those sequence of variables and terminal symbols that can be obtained from the production rules of the grammar.

Show that, if $\mathcal{G}$ is a context-free grammar in Chomsky normal form and ω is a non-empty sentential form only composed of non-terminal symbols ($\omega \in V^* \setminus \{\epsilon\}$), then ω can be obtained by applying $r(\omega) = |\omega| - 1$ production rules.

Exercise 5.3 (Pumping Lemma)

Are the following languages context-free?

(a) $L_1 := \{1^n 0^m 1^{nm} \mid n, m \geq 0\}$.

(b) $L_2 := \{a^{n^2} \mid n \geq 0\}$