

# Robot Mapping

## Graph-Based SLAM with Landmarks

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# Graph-Based SLAM (Chap. 15)

- Use a **graph** to represent the problem
- Every **node** in the graph corresponds to a pose of the robot during mapping
- Every **edge** between two nodes corresponds to a spatial constraint between them
- **Graph-Based SLAM:** Build the graph and find a node configuration that minimize the error introduced by the constraints

# The Graph

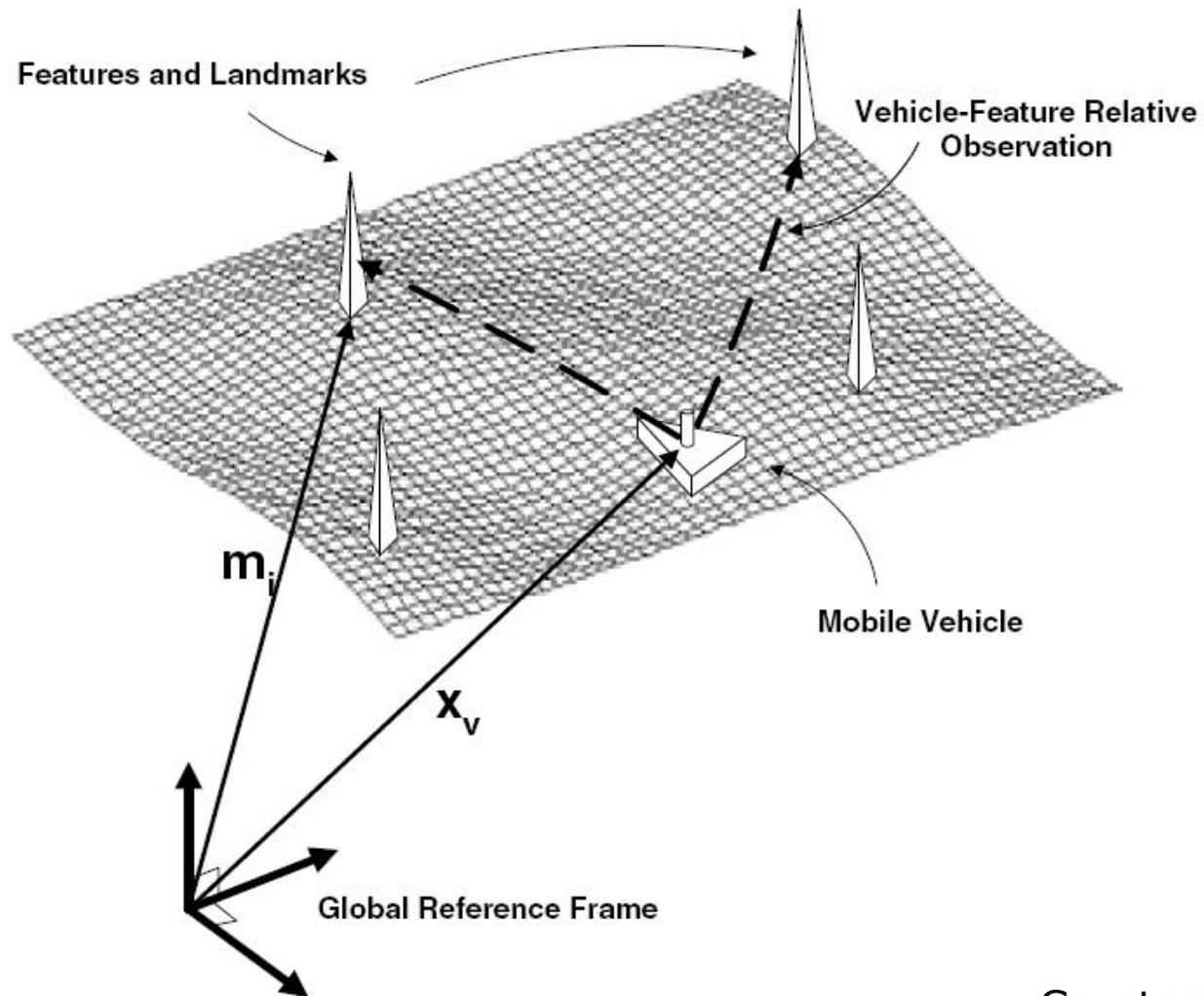
## So far:

- Vertices for robot poses  $(x, y, \theta)$
- Edges for virtual observations (transformations) between robot poses

## Topic today:

- How to deal with landmarks

# Landmark-Based SLAM

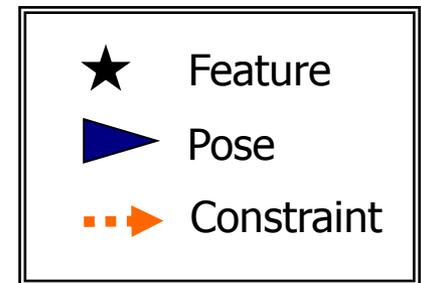
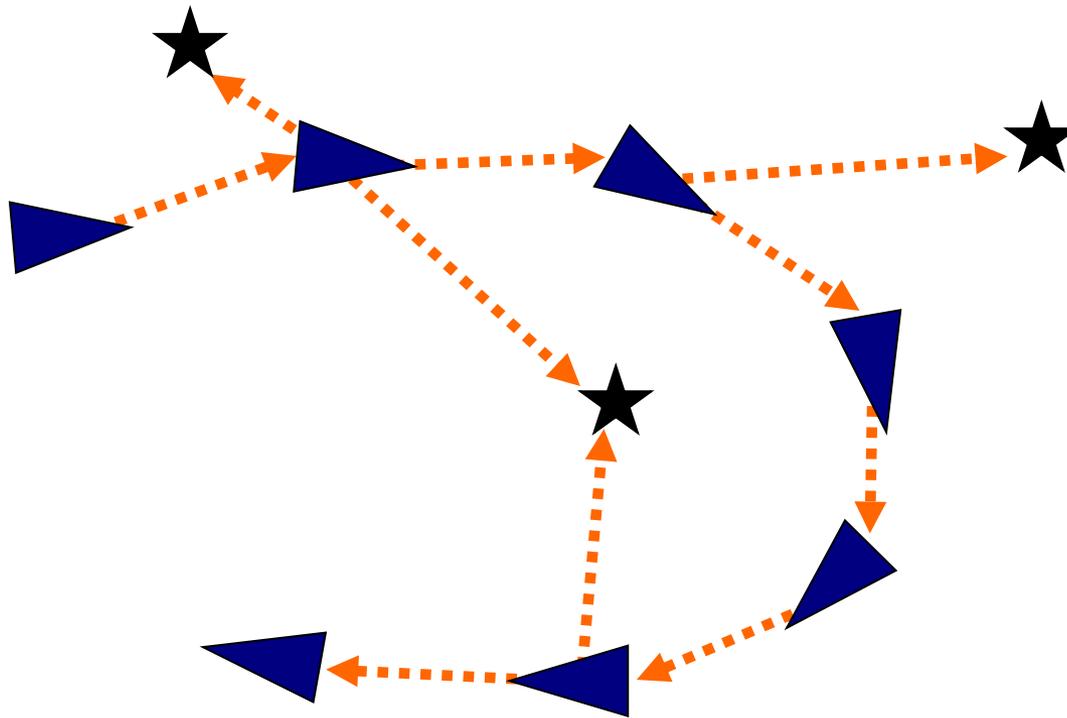


# Real Landmark Map Example



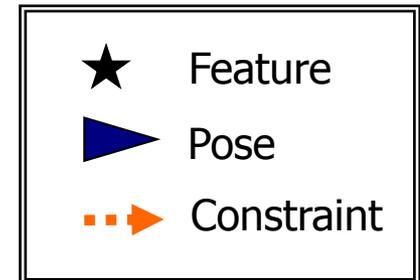
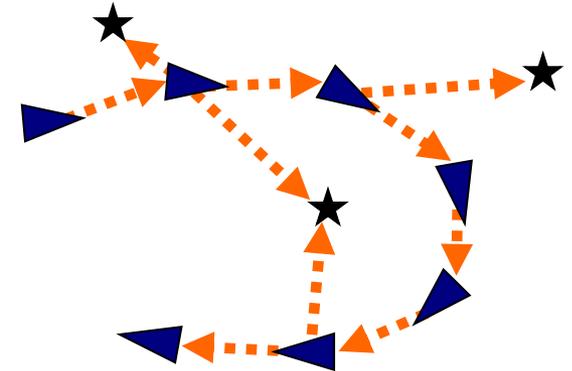
Image courtesy: E. Nebot

# The Graph with Landmarks



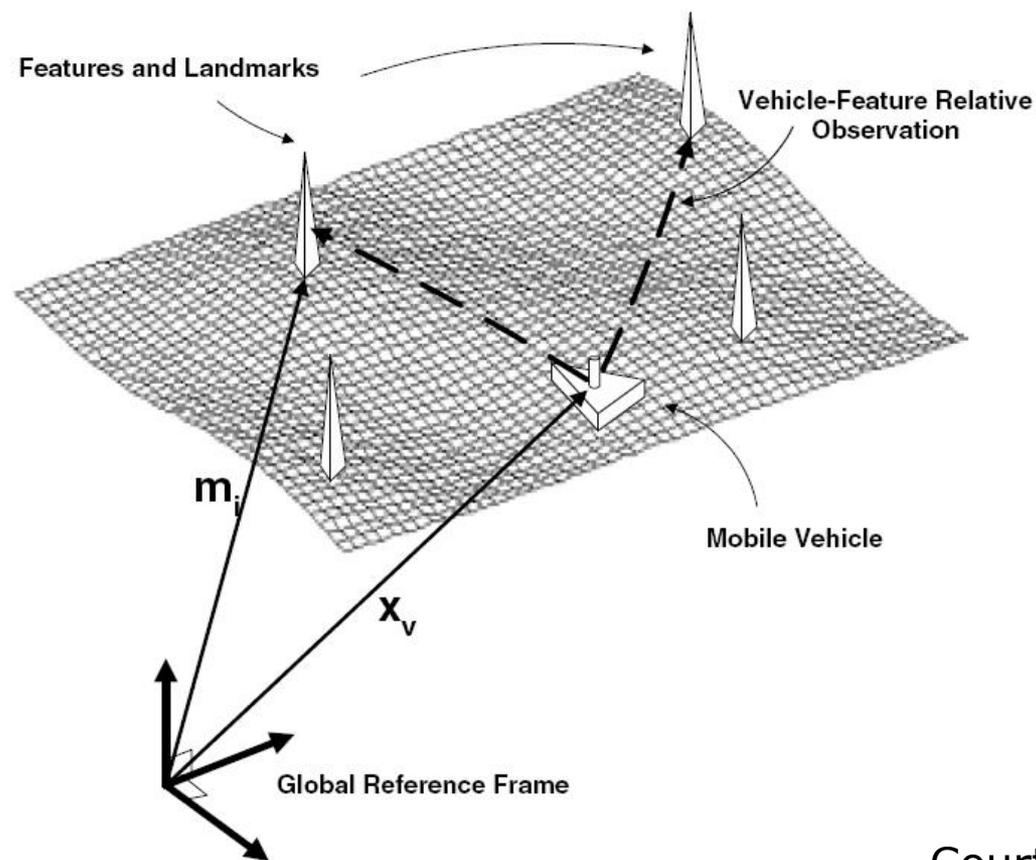
# The Graph with Landmarks

- **Nodes** can represent:
  - Robot poses
  - Landmark locations
- **Edges** can represent:
  - Landmark observations
  - Odometry measurements
- The minimization optimizes the landmark locations and robot poses



# 2D Landmarks

- Landmark is a  $(x, y)$ -point in the world
- Relative observation in  $(x, y)$



# Landmarks Observation

- Expected observation (x-y sensor)

$$\hat{\mathbf{z}}_{ij}(\mathbf{x}_i, \mathbf{x}_j) = \mathbf{R}_i^T (\mathbf{x}_j - \mathbf{t}_i)$$

robot      landmark                      robot translation





# Bearing Only Observations

- Observation function

$$\hat{\mathbf{z}}_{ij}(\mathbf{x}_i, \mathbf{x}_j) = \text{atan} \frac{(\mathbf{x}_j - \mathbf{t}_i).y}{(\mathbf{x}_j - \mathbf{t}_i).x} - \theta_i$$

↑ robot    ↑ landmark    ↑ robot-landmark angle    ↑ robot orientation

- Error function

$$\mathbf{e}_{ij}(\mathbf{x}_i, \mathbf{x}_j) = \text{atan} \frac{(\mathbf{x}_j - \mathbf{t}_i).y}{(\mathbf{x}_j - \mathbf{t}_i).x} - \theta_i - \mathbf{z}_j$$

# The Rank of the Matrix $\mathbf{H}$

- What is the rank of  $\mathbf{H}_{ij}$  for a 2D landmark-pose constraint?

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  - $\mathbf{H}_{ij}$  cannot have more than rank 2  
 $\text{rank}(A^T A) = \text{rank}(A^T) = \text{rank}(A)$

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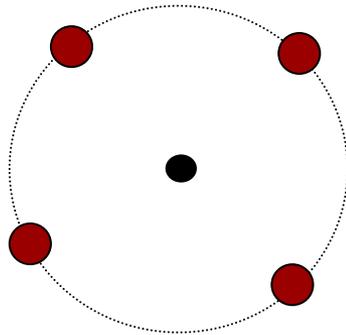
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- What is the rank of  $\mathbf{H}_{ij}$  for a bearing-only constraint?
  - The blocks of  $\mathbf{J}_{ij}$  are a 1x3 matrices
  - $\mathbf{H}_{ij}$  has rank 1

# Where is the Robot?

- Robot observes one landmark  $(x,y)$
- Where can the robot be relative to the landmark?

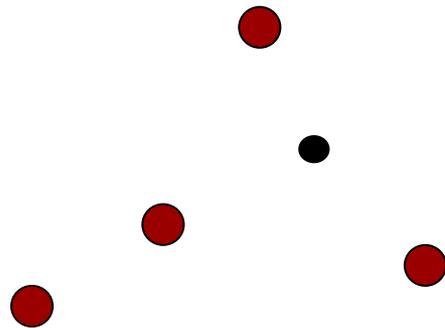


The robot can be somewhere on a circle around the landmark

It is a 1D solution space (constrained by the distance and the robot's orientation)

# Where is the Robot?

- Robot observes one landmark (bearing-only)
- Where can the robot be relative to the landmark?



The robot can be anywhere  
in the x-y plane

It is a 2D solution space  
(constrained by the robot's  
orientation)

# Rank

- In landmark-based SLAM, the system can be under-determined
- The rank of  $\mathbf{H}$  is **less or equal** to the sum of the ranks of the constraints
- To determine a **unique solution**, the system must have **full rank**

# Questions

- The rank of  $\mathbf{H}$  is **less or equal** to the sum of the ranks of the constraints
- To determine a **unique solution**, the system must have **full rank**
- **Questions:**
  - How many 2D landmark observations are needed to resolve for a robot pose?
  - How many bearing-only observations are needed to resolve for a robot pose?

# Under-Determined Systems

- No guarantee for a full rank system
  - Landmarks may be observed only once
  - Robot might have no odometry
- We can still deal with these situations by adding a “damping” factor to  $\mathbf{H}$
- Instead of solving  $\mathbf{H}\Delta\mathbf{x} = -\mathbf{b}$ , we solve

$$(\mathbf{H} + \lambda\mathbf{I})\Delta\mathbf{x} = -\mathbf{b}$$

**What is the effect of that?**

$$(H + \lambda I) \Delta x = -b$$

- Damping factor for  $H$
- $(H + \lambda I) \Delta x = -b$
- The damping factor  $\lambda I$  makes the system positive definite
- Weighted sum of Gauss Newton and Steepest Descent

# Simplified Levenberg Marquardt

- Damping to regulate the convergence using backup/restore actions

```
x: the initial guess
while (! converged)
     $\lambda = \lambda_{\text{init}}$ 
    <H, b> = buildLinearSystem(x);
    E = error(x)
    xold = x;
     $\Delta\mathbf{x}$  = solveSparse( (H +  $\lambda$  I)  $\Delta\mathbf{x}$  = -b);
    x +=  $\Delta\mathbf{x}$ ;
    If (E < error(x)) {
        x = xold;
         $\lambda$  *= 2;
    } else {  $\lambda$  /= 2; }
```

# Bundle Adjustment

- 3D reconstruction based on images taken at different viewpoints
- Minimizes the reprojection error
- Often uses Levenberg Marquardt
- Developed in photogrammetry during the 1950ies

# Summary

- Graph-Based SLAM for landmarks
- The rank of  $\mathbf{H}$  matters
- Levenberg Marquardt for optimization

# Literature

## **Bundle Adjustment:**

- Triggs et al. “Bundle Adjustment — A Modern Synthesis”

# Slide Information

- These slides have been created by Cyrill Stachniss as part of the robot mapping course taught in 2012/13 and 2013/14. I created this set of slides partially extending existing material of Giorgio Grisetti and myself.
- I tried to acknowledge all people that contributed image or video material. In case I missed something, please let me know. If you adapt this course material, please make sure you keep the acknowledgements.
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