

Robot Mapping

Graph-Based SLAM with Landmarks

**Gian Diego Tipaldi, Luciano Spinello,
Wolfram Burgard**

Graph-Based SLAM (Chap. 15)

- Use a **graph** to represent the problem
- Every **node** in the graph corresponds to a pose of the robot during mapping
- Every **edge** between two nodes corresponds to a spatial constraint between them
- **Graph-Based SLAM:** Build the graph and find a node configuration that minimize the error introduced by the constraints

The Graph

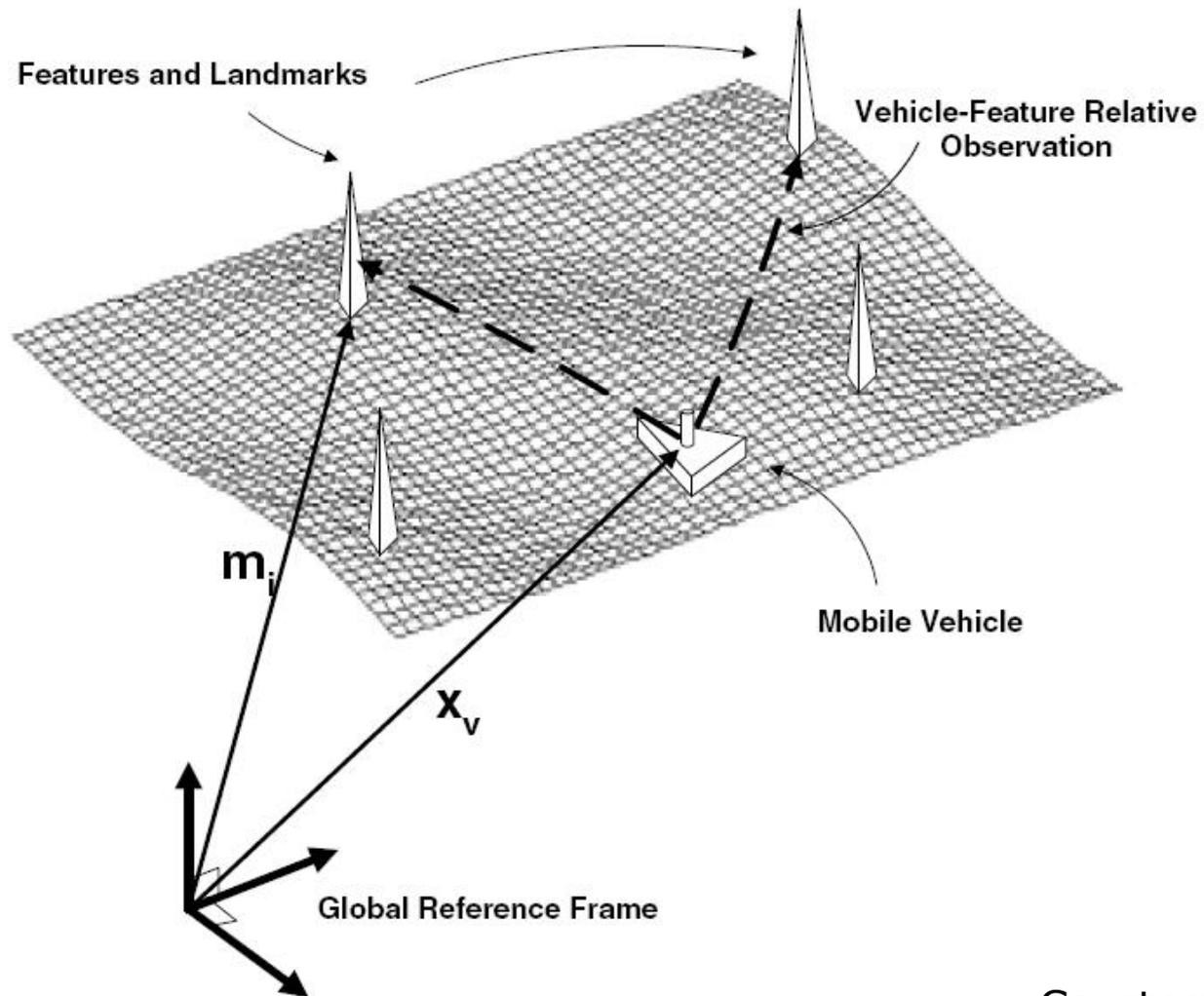
So far:

- Vertices for robot poses (x, y, θ)
- Edges for virtual observations (transformations) between robot poses

Topic today:

- How to deal with landmarks

Landmark-Based SLAM

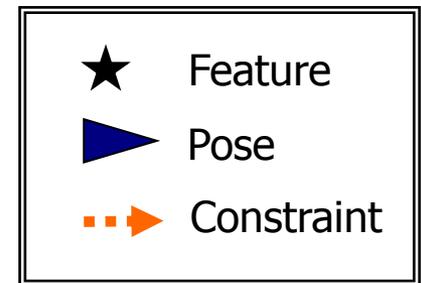
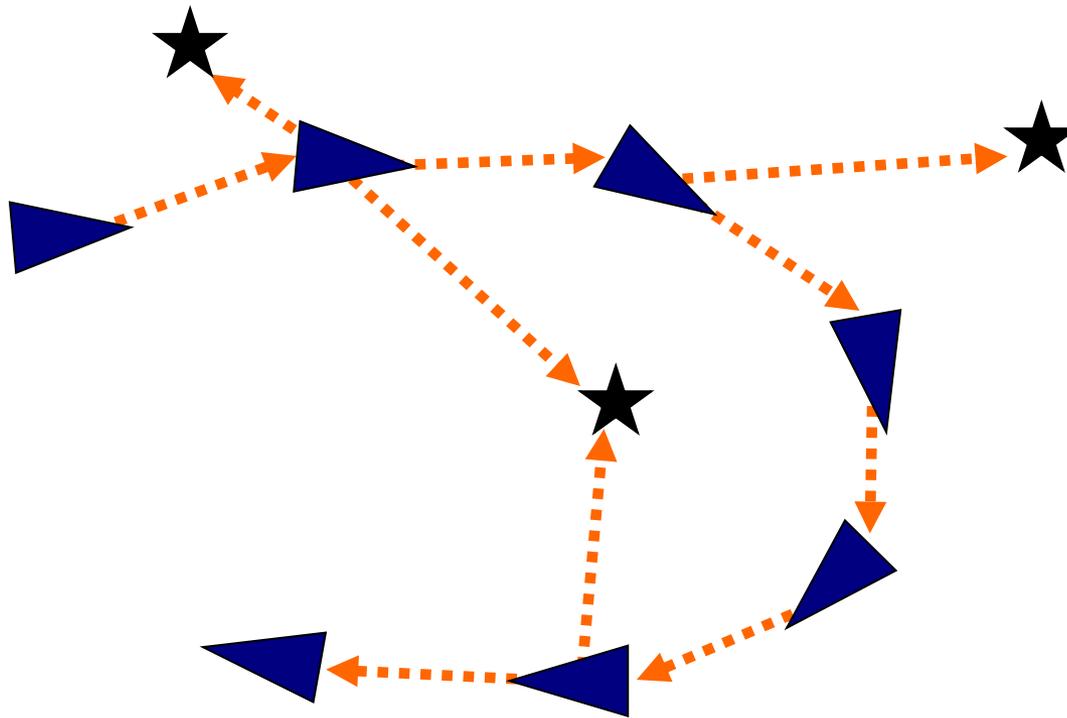


Real Landmark Map Example



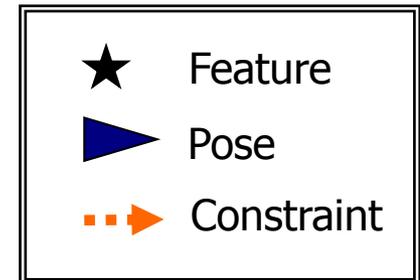
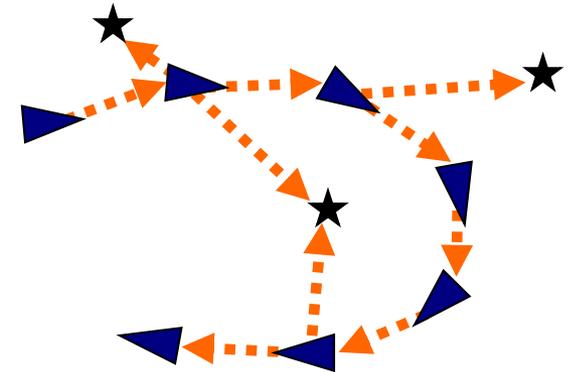
Image courtesy: E. Nebot

The Graph with Landmarks



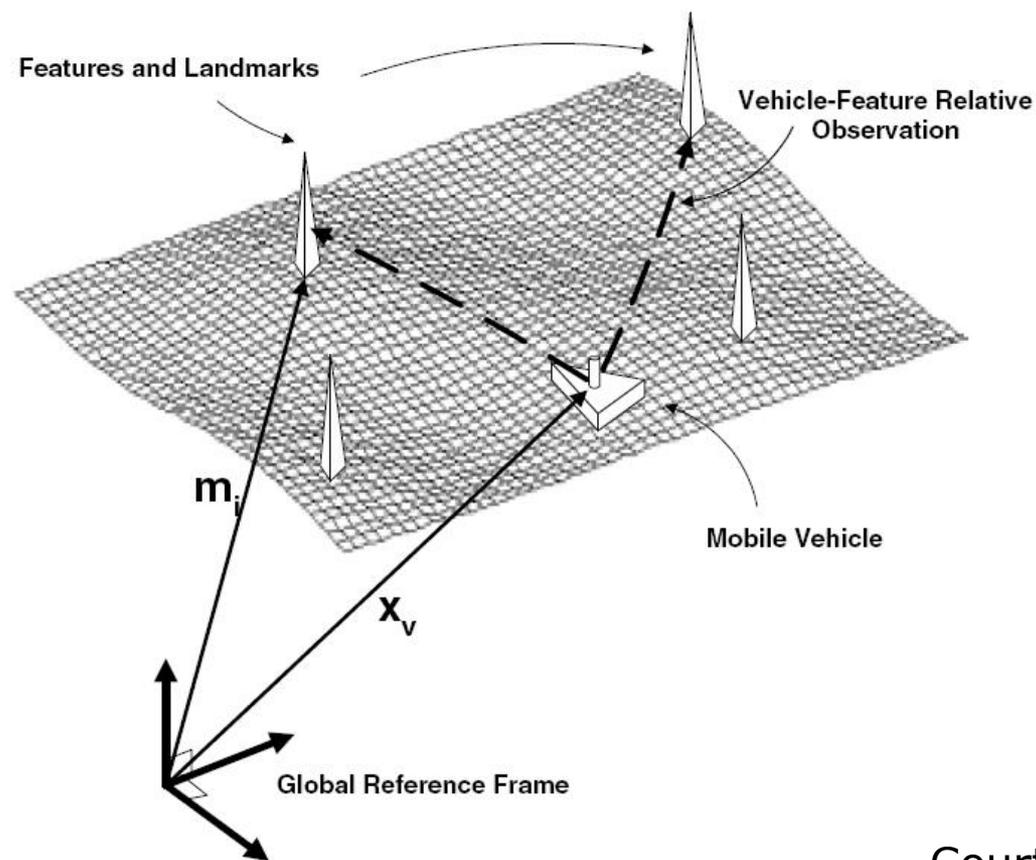
The Graph with Landmarks

- **Nodes** can represent:
 - Robot poses
 - Landmark locations
- **Edges** can represent:
 - Landmark observations
 - Odometry measurements
- The minimization optimizes the landmark locations and robot poses



2D Landmarks

- Landmark is a (x, y) -point in the world
- Relative observation in (x, y)



Bearing Only Observations

- A landmark is still a 2D point
- The robot observe only the bearing towards the landmark
- Observation function

$$\hat{\mathbf{z}}_{ij}(\mathbf{x}_i, \mathbf{x}_j) = \text{atan} \frac{(\mathbf{x}_j - \mathbf{t}_i).y}{(\mathbf{x}_j - \mathbf{t}_i).x} - \theta_i$$

↑ robot ↑ landmark ↑ robot-landmark angle ↑ robot orientation

Bearing Only Observations

- Observation function

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- Error function

$$\mathbf{e}_{ij}(\mathbf{x}_i, \mathbf{x}_j) = \text{atan} \frac{(\mathbf{x}_j - \mathbf{t}_i).y}{(\mathbf{x}_j - \mathbf{t}_i).x} - \theta_i - \mathbf{z}_j$$

The Rank of the Matrix \mathbf{H}

- What is the rank of \mathbf{H}_{ij} for a 2D landmark-pose constraint?

The Rank of the Matrix H

- What is the rank of \mathbf{H}_{ij} for a 2D landmark-pose constraint?
 - The blocks of \mathbf{J}_{ij} are a 2x3 matrices
 - \mathbf{H}_{ij} cannot have more than rank 2
 $\text{rank}(A^T A) = \text{rank}(A^T) = \text{rank}(A)$

The Rank of the Matrix \mathbf{H}

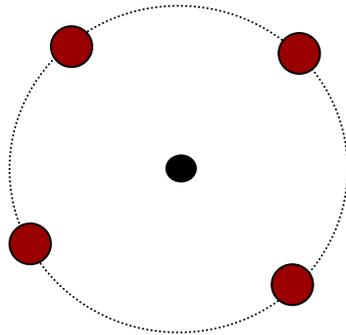
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- What is the rank of \mathbf{H}_{ij} for a bearing-only constraint?
 - The blocks of \mathbf{J}_{ij} are a 1x3 matrices
 - \mathbf{H}_{ij} has rank 1

Where is the Robot?

- Robot observes one landmark (x,y)
- Where can the robot be relative to the landmark?

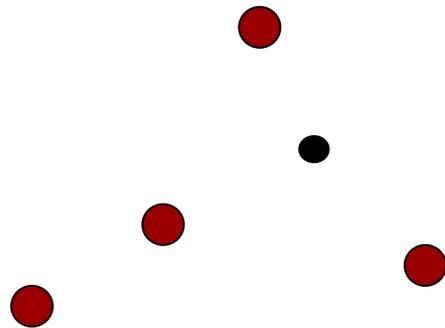


The robot can be somewhere on a circle around the landmark

It is a 1D solution space (constrained by the distance and the robot's orientation)

Where is the Robot?

- Robot observes one landmark (bearing-only)
- Where can the robot be relative to the landmark?



The robot can be anywhere
in the x-y plane

It is a 2D solution space
(constrained by the robot's
orientation)

Rank

- In landmark-based SLAM, the system can be under-determined
- The rank of \mathbf{H} is **less or equal** to the sum of the ranks of the constraints
- To determine a **unique solution**, the system must have **full rank**

Questions

- The rank of \mathbf{H} is **less or equal** to the sum of the ranks of the constraints
- To determine a **unique solution**, the system must have **full rank**
- **Questions:**
 - How many 2D landmark observations are needed to resolve for a robot pose?
 - How many bearing-only observations are needed to resolve for a robot pose?

Under-Determined Systems

- No guarantee for a full rank system
 - Landmarks may be observed only once
 - Robot might have no odometry
- We can still deal with these situations by adding a “damping” factor to \mathbf{H}
- Instead of solving $\mathbf{H}\Delta\mathbf{x} = -\mathbf{b}$, we solve

$$(\mathbf{H} + \lambda\mathbf{I})\Delta\mathbf{x} = -\mathbf{b}$$

What is the effect of that?

$$(H + \lambda I) \Delta x = -b$$

- Damping factor for H
- $(H + \lambda I) \Delta x = -b$
- The damping factor λI makes the system positive definite
- Weighted sum of Gauss Newton and Steepest Descent

Simplified Levenberg Marquardt

- Damping to regulate the convergence using backup/restore actions

```
x: the initial guess
while (! converged)
     $\lambda = \lambda_{\text{init}}$ 
    <H, b> = buildLinearSystem(x);
    E = error(x)
    xold = x;
     $\Delta\mathbf{x}$  = solveSparse( (H +  $\lambda$  I)  $\Delta\mathbf{x}$  = -b);
    x +=  $\Delta\mathbf{x}$ ;
    If (E < error(x)) {
        x = xold;
         $\lambda$  *= 2;
    } else {  $\lambda$  /= 2; }
```

Bundle Adjustment

- 3D reconstruction based on images taken at different viewpoints
- Minimizes the reprojection error
- Often uses Levenberg Marquardt
- Developed in photogrammetry during the 1950ies

Summary

- Graph-Based SLAM for landmarks
- The rank of \mathbf{H} matters
- Levenberg Marquardt for optimization

Literature

Bundle Adjustment:

- Triggs et al. “Bundle Adjustment — A Modern Synthesis”

Slide Information

- These slides have been created by Cyrill Stachniss as part of the robot mapping course taught in 2012/13 and 2013/14. I created this set of slides partially extending existing material of Giorgio Grisetti and myself.
- I tried to acknowledge all people that contributed image or video material. In case I missed something, please let me know. If you adapt this course material, please make sure you keep the acknowledgements.
- Feel free to use and change the slides. If you use them, I would appreciate an acknowledgement as well. To satisfy my own curiosity, I appreciate a short email notice in case you use the material in your course.
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http://www.youtube.com/playlist?list=PLgnQpQtFTOGQrZ4O5QzbIHgl3b1JHimN_&feature=g-list