Albert-Ludwigs-Universität Freiburg, Institut für Informatik
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Lecture: Robot Mapping
Winter term 2015

Sheet 3

Topic: Extended Kalman Filter SLAM

Submission deadline: Nov. 9 2014
Submit to: `robotmappingtutors@informatik.uni-freiburg.de`

Exercise 1: Bayes Filter and EKF

(a) Describe briefly the two main steps of the Bayes filter in your own words.

(b) Describe briefly the meaning of the following probability density functions: $p(x_t \mid u_t, x_{t-1})$, $p(z_t \mid x_t)$, and $\mathrm{bel}(x_t)$, which are processed by the Bayes filter.

(c) Specify the (normal) distributions that correspond to the above mentioned three terms in EKF SLAM.

(d) Explain in a few sentences all of the components of the EKF SLAM algorithm, i. e., μ_t, Σ_t, g, G_t^x, G_t, R_t^x, R_t, h, H_t, Q_t, K_t and why they are needed. Specify the dimensionality of these components.

Exercise 2: Jacobians

(a) Derive the Jacobian matrix G_t^x of the noise-free motion function g with respect to the pose of the robot. Use the odometry motion model as in exercise sheet 1:

$$\begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} = \begin{pmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{pmatrix} + \begin{pmatrix} \delta_{trans}\cos(\theta_{t-1} + \delta_{rot1}) \\ \delta_{trans}\sin(\theta_{t-1} + \delta_{rot1}) \\ \delta_{rot1} + \delta_{rot2} \end{pmatrix}.$$

Do not use Octave for this part of the exercise.

(b) Derive the Jacobian matrix ${}^{\mathrm{low}}H_t^i$ of the noise-free sensor function h corresponding to the i^{th} measurement:

$$h(\bar{\mu}_t, j) = z_t^i = \begin{pmatrix} r_t^i \\ \phi_t^i \end{pmatrix} = \begin{pmatrix} \sqrt{(\bar{\mu}_{j,x} - \bar{\mu}_{t,x})^2 + (\bar{\mu}_{j,y} - \bar{\mu}_{t,y})^2} \\ \mathrm{atan2}(\bar{\mu}_{j,y} - \bar{\mu}_{t,y}, \bar{\mu}_{j,x} - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{pmatrix},$$

where $(\bar{\mu}_{j,x}, \bar{\mu}_{j,y})^T$ is the pose of the j^{th} landmark, $(\bar{\mu}_{t,x}, \bar{\mu}_{t,y}, \bar{\mu}_{t,\theta})^T$ is the pose of the robot at time t, and r_t^i and ϕ_t^i are respectively the observed range and bearing of the landmark. Do not use Octave for this part of the exercise.

Hint: use $\frac{\partial}{\partial x}\mathrm{atan2}(y, x) = \frac{-y}{x^2+y^2}$, and $\frac{\partial}{\partial y}\mathrm{atan2}(y, x) = \frac{x}{x^2+y^2}$.