Basic Probability Rules

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1 Basic Axioms

\[ 0 \leq p(x) \leq 1 \]  \hspace{1cm} (1)

\[ p(true) = 1 \text{ and } p(false) = 0 \]  \hspace{1cm} (2)

\[ p(x \lor y) = p(x) + p(y) - p(x \land y) \]  \hspace{1cm} (3)

2 Complement Probability

\[ p(\neg x) = 1 - p(x). \]  \hspace{1cm} (4)

3 Product Rule

The following equation is called the product rule

\[ p(x, y) = p(x \mid y) \cdot p(y) \]  \hspace{1cm} (5)

\[ = p(y \mid x) \cdot p(x). \]  \hspace{1cm} (6)

4 Independence

If \( x \) and \( y \) are independent, we have

\[ p(x, y) = p(x) \cdot p(y). \]  \hspace{1cm} (7)

5 Bayes’ Rule

The Bayes’ rule, which is frequently used in this thesis, is given by

\[ p(x \mid y) = \frac{p(y \mid x) \cdot p(x)}{p(y)}. \]  \hspace{1cm} (8)

The denominator is a normalizing constant that ensures that the posterior of the left hand side adds up to 1 over all possible values. Thus, we often write

\[ p(x \mid y) = \eta \cdot p(y \mid x) \cdot p(x). \]  \hspace{1cm} (9)

In case the background knowledge \( e \) is given, Bayes’ rule turns into

\[ p(x \mid y, e) = \frac{p(y \mid x, e) \cdot p(x \mid e)}{p(y \mid e)}. \]  \hspace{1cm} (10)
6 Marginalization

The marginalization rule is the following equation

\[ p(x) = \int y p(x, y) \, dy. \]  \hfill (11)

In the discrete case, the integral turns into a sum

\[ p(x) = \sum_y p(x, y). \]  \hfill (12)

7 Law of Total Probability

The law of total probability is a variant of the marginalization rule, which can be derived using the product rule

\[ p(x) = \int y p(x \mid y) \cdot p(y) \, dy, \]  \hfill (13)

and the corresponding sum for the discrete case

\[ p(x) = \sum_y p(x \mid y) \cdot p(y). \]  \hfill (14)

8 Markov Assumption

The Markov assumption (also called Markov property) characterizes the fact that a variable \( x_t \) depends only on its direct predecessor state \( x_{t-1} \) and not on \( x_{t'} \) with \( t' < t - 1 \)

\[ p(x_t \mid x_{1:t-1}) = p(x_t \mid x_{t-1}). \]  \hfill (15)