Robot Mapping

Summary on the Kalman Filter & Friends: KF, EKF, UKF, EIF, SEIF

Gian Diego Tipaldi, Wolfram Burgard
Three Main SLAM Paradigms

- Kalman filter
- Particle filter
- Graph-based
Kalman Filter & Its Friends

Kalman filter

Particle filter

Graph-based

Kalman filter

Extended Kalman Filter

Unscented Kalman Filter

Extended Information Filter

Sparse Extended Information Filter
Kalman Filter Algorithm

1: \textbf{Kalman}\_\texttt{filter}(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t):

2: \begin{align*}
    \bar{\mu}_t &= A_t \mu_{t-1} + B_t u_t \\
    \bar{\Sigma}_t &= A_t \Sigma_{t-1} A^T_t + R_t
\end{align*}

prediction

3: \begin{align*}
    K_t &= \bar{\Sigma}_t C^T_t (C_t \bar{\Sigma}_t C^T_t + Q_t)^{-1} \\
    \mu_t &= \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t) \\
    \Sigma_t &= (I - K_t C_t) \bar{\Sigma}_t
\end{align*}

correction

5: \text{return } \mu_t, \Sigma_t

6: \begin{align*}
    \Sigma_t &= (I - K_t C_t) \bar{\Sigma}_t
\end{align*}
Non-linear Dynamic Systems

- Most realistic problems in robotics involve nonlinear functions

\[
x_t = A_t x_{t-1} + B_t u_t + \epsilon_t
\]

\[
z_t = C_t x_t + \delta_t
\]

\[
x_t = g(u_t, x_{t-1}) + \epsilon_t
\]

\[
z_t = h(x_t) + \delta_t
\]

requires linearization

\[\rightarrow\] EKF
KF vs. EKF

- EKF is an extension of the KF
- Approach to handle the non-linearities
- Performs local linearizations
- Works well in practice for moderate non-linearities and uncertainty
EKF for SLAM

\[
\begin{pmatrix}
    x_R \\
    m_1 \\
    \vdots \\
    m_n
\end{pmatrix}
\end{pmatrix}

\begin{pmatrix}
    \sum x_R x_R \\
    \sum m_1 x_R \\
    \vdots \\
    \sum m_n x_R
\end{pmatrix}

\begin{pmatrix}
    \sum x_R m_1 & \cdots & \sum x_R m_n \\
    \sum m_1 m_1 & \cdots & \sum m_1 m_n \\
    \vdots & \ddots & \vdots \\
    \sum m_n m_1 & \cdots & \sum m_n m_n
\end{pmatrix}
EKF SLAM

Map

Correlation matrix

Courtesy: M. Montemerlo
EKF SLAM

Map

Correlation matrix

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EKF SLAM

Map

Correlation matrix

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EKF-SLAM Properties

- In the limit, the landmark estimates become **fully correlated**
EKF-SLAM Complexity

- Cubic complexity only on the measurement dimensionality
- Cost per step: dominated by the number of landmarks: $O(n^2)$
- Memory consumption: $O(n^2)$
- The EKF becomes computationally intractable for large maps!
Unscented Kalman Filter (UKF)

UKF Motivation

- Kalman filter requires linear models
- EKF linearizes via Taylor expansion

Is there a better way to linearize?

Unscented Transform

Unscented Kalman Filter (UKF)
Taylor Approximation (EKF)

Linearization of the non-linear function through Taylor expansion
Unscented Transform

Compute a set of (so-called) sigma points
Unscented Transform

Transform each sigma point through the non-linear motion and measurement functions
Unscented Transform

Reconstruct a Gaussian from the transformed and weighted points
UKF vs. EKF

- Same results as EKF for linear models
- Better approximation than EKF for non-linear models
- Differences often “somewhat small”
- No Jacobians needed for the UKF
- Same complexity class
- Slightly slower than the EKF
EIF: Two Parameterizations for a Gaussian Distribution

moments

\[ \Sigma = \Omega^{-1} \]
\[ \mu = \Omega^{-1} \xi \]

covariance matrix
mean vector

canonical

\[ \Omega = \Sigma^{-1} \]
\[ \xi = \Sigma^{-1} \mu \]

information matrix
information vector
Extended Information Filter

- The EIF is the EKF in information form
- Instead of the moments $\Sigma, \mu$ the canonical form is maintained using $\Omega, \xi$
- Conversion between information for and canonical form is expensive
- EIF has the same expressiveness than the EKF
EIF vs. EKF

- Complexity of the prediction and corrections steps differs
  - KF: efficient prediction, slow correction
  - IF: slow prediction, efficient correction
- “The application determines the filter”
- In practice, the EKF is more popular than the EIF
Motivation for SEIF SLAM

Gaussian estimate (map & pose) normalized covariance matrix normalized information matrix

Courtesy: Thrun, Burgard, Fox
Keep the Links Between in the Information Matrix Bounded

normalized information matrix

link

robot

features active passive

Courtesy: Thrun, Burgard, Fox
Four Steps of SEIF SLAM

1. Motion update
2. Measurement update
3. Update of the state estimate
4. Sparsification
Efficiency of SEIF SLAM

- Maintains the robot-landmark links only for a small set of landmarks at a time
- Removes robot-landmark links by sparsification (equal to assuming conditional independence)
- This also bounds the number of landmark-landmark links
- Exploits the sparsity of the information matrix in all computations
SEIF SLAM vs. EKF SLAM

- SEIFs are an efficient **approximation** of the EIF for the SLAM problem
- Neglects links by sparsification
- **Constant time** updates of the filter (for known correspondences)
- **Linear memory** complexity
- **Inferior quality** compared to EKF SLAM
Summary

- KFs deal differently with non-linear motion and measurement functions
- KF, EKF, UKF, EIF suffer from complexity issues for large maps
- SEIF approximations lead to sub-quadratic memory and runtime complexity
- All filters presented so far, require Gaussian distributions
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I tried to acknowledge all people that contributed image or video material. In case I missed something, please let me know. If you adapt this course material, please make sure you keep the acknowledgements.

Feel free to use and change the slides. If you use them, I would appreciate an acknowledgement as well. To satisfy my own curiosity, I appreciate a short email notice in case you use the material in your course.

My video recordings are available through YouTube: http://www.youtube.com/playlist?list=PLgnQpQtFTOGQrZ4O5QzbIHgl3b1JHimN_&feature=g-list

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