Robot Mapping

FastSLAM – Feature-Based SLAM with Particle Filters

Gian Diego Tipaldi, Wolfram Burgard

Particle Filter

- Non-parametric recursive Bayes filter
- Posterior is represented by a set of weighted samples
- Can model arbitrary distributions
- Works well in low-dimensional spaces
- 3-Step procedure
 - Sampling from proposal
 - Importance Weighting
 - Resampling

Particle Filter Algorithm

 Sample the particles from the proposal distribution

$$x_t^{[j]} \sim \pi(x_t \mid \ldots)$$

2. Compute the importance weights

$$w_t^{[j]} = \frac{target(x_t^{[j]})}{proposal(x_t^{[j]})}$$

1. Resampling: Draw sample i with probability $w_t^{[i]}$ and repeat J times

Particle Representation

A set of weighted samples

$$\mathcal{X} = \left\{ \left\langle x^{[i]}, w^{[i]} \right\rangle \right\}_{i=1,\dots,N}$$

- Think of a sample as one hypothesis about the state
- For feature-based SLAM:

$$x = (x_{1:t}, m_{1,x}, m_{1,y}, \dots, m_{M,x}, m_{M,y})^T$$
poses landmarks

Dimensionality Problem

Particle filters are effective in low dimensional spaces. The likely regions of the state space need to be covered with samples.

Higher dimensions -> more samples.

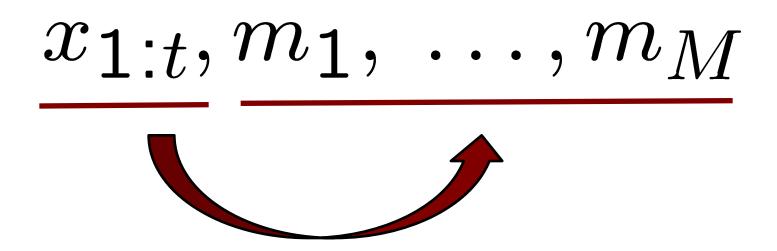
$$x = (x_{1:t}, m_{1,x}, m_{1,y}, \dots, m_{M,x}, m_{M,y})^T$$

high-dimensional

Can We Exploit Dependencies Between the Different Dimensions of the State Space?

$x_{1:t}, m_1, \ldots, m_M$

If We Know the Poses of the Robot, Mapping is Easy!



Key Idea

$$x_{1:t}, m_1, \ldots, m_M$$

If we use the particle set only to model the robot's path, each sample is a path hypothesis. For each sample, we can compute an individual map of landmarks.

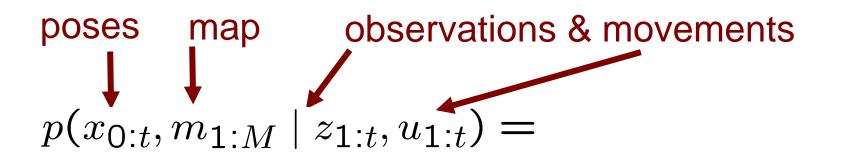
Rao-Blackwellization

 Factorization to exploit dependencies between variables:

$$p(a,b) = p(b \mid a) p(a)$$

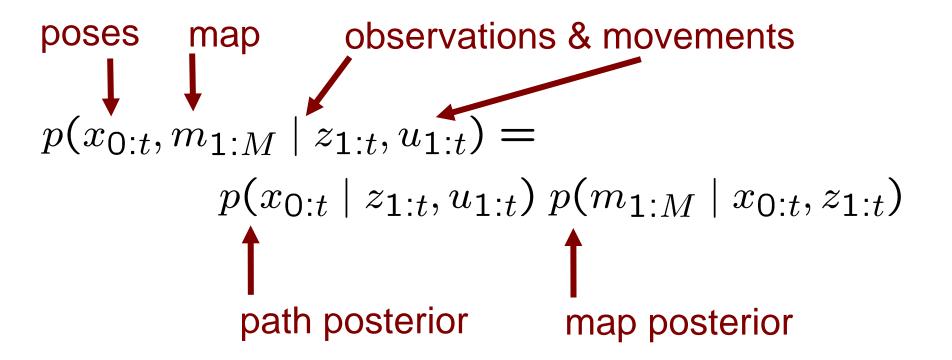
 If p(b | a) can be computed in closed form, represent only p(a) with samples and compute p(b | a) for every sample

Factorization of the SLAM posterior



First introduced for SLAM by Murphy in 1999

Factorization of the SLAM posterior



First introduced for SLAM by Murphy in 1999

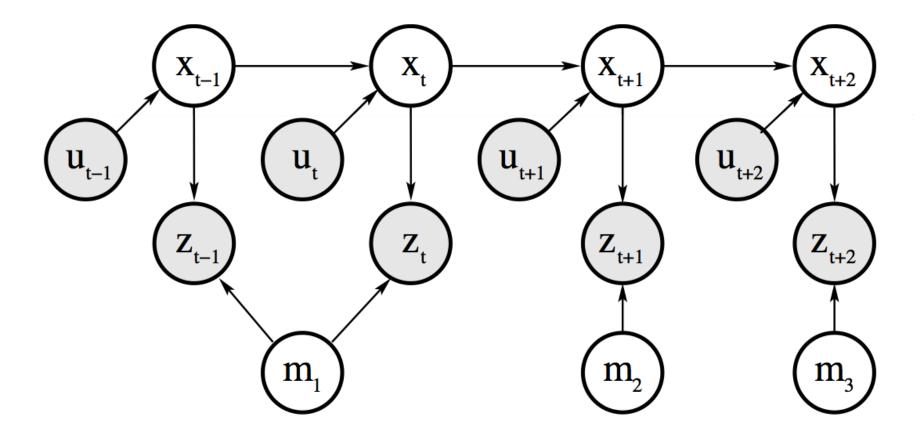
Factorization of the SLAM posterior

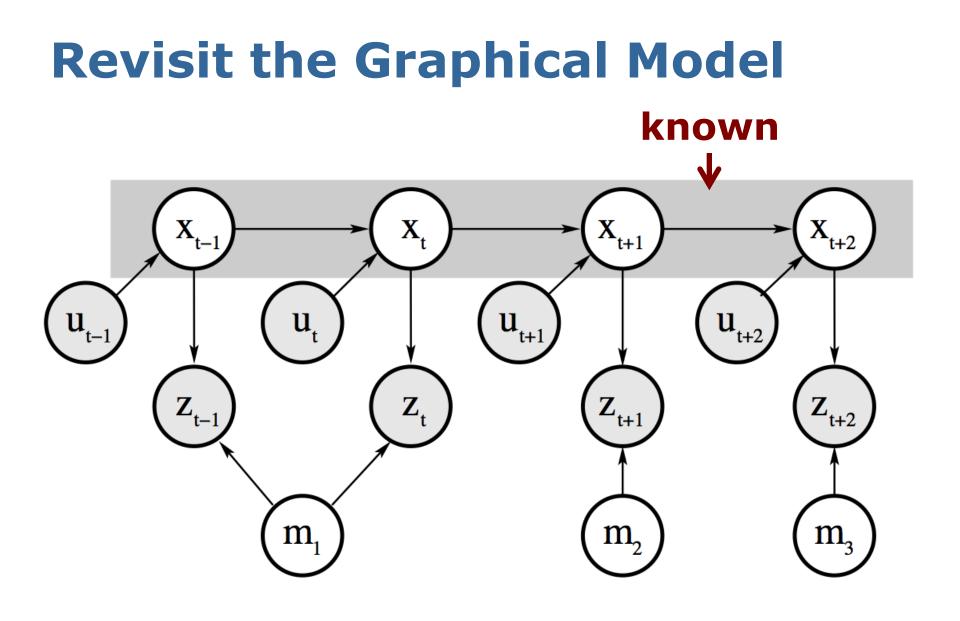
 $p(x_{0:t}, m_{1:M} \mid z_{1:t}, u_{1:t}) = p(x_{0:t} \mid z_{1:t}, u_{1:t}) p(m_{1:M} \mid x_{0:t}, z_{1:t})$

How to compute this term efficiently?

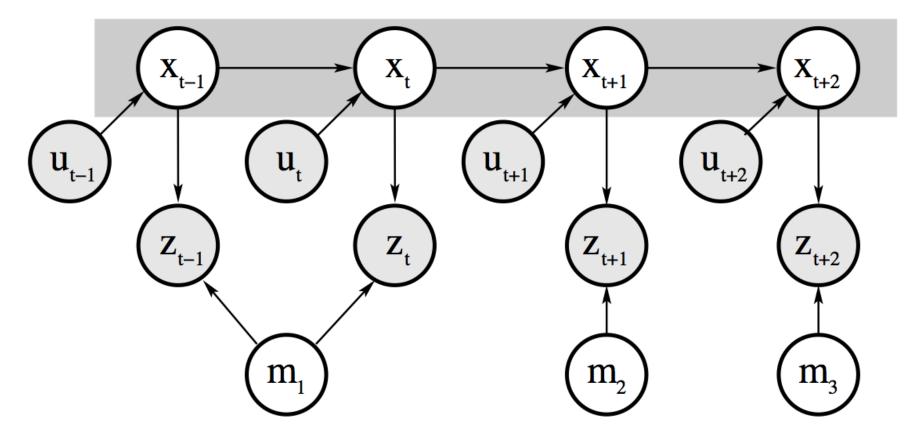
First introduced for SLAM by Murphy in 1999

Revisit the Graphical Model





Landmarks are Conditionally Independent Given the Poses



Landmark variables are all disconnected (i.e. independent) given the robot's path

Factorization of the SLAM posterior

 $p(x_{0:t}, m_{1:M} \mid z_{1:t}, u_{1:t}) = p(x_{0:t} \mid z_{1:t}, u_{1:t}) p(m_{1:M} \mid x_{0:t}, z_{1:t})$

Landmarks are conditionally independent given the poses

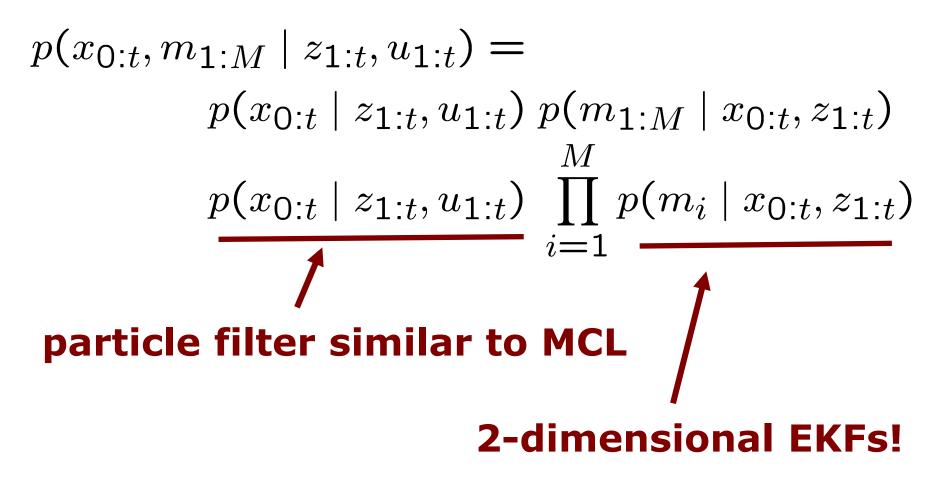
Factorization of the SLAM posterior

 $p(x_{0:t}, m_{1:M} \mid z_{1:t}, u_{1:t}) =$ $p(x_{0:t} \mid z_{1:t}, u_{1:t}) p(m_{1:M} \mid x_{0:t}, z_{1:t})$ $p(x_{0:t} \mid z_{1:t}, u_{1:t}) \prod_{i=1}^{M} p(m_i \mid x_{0:t}, z_{1:t})$

Factorization of the SLAM posterior

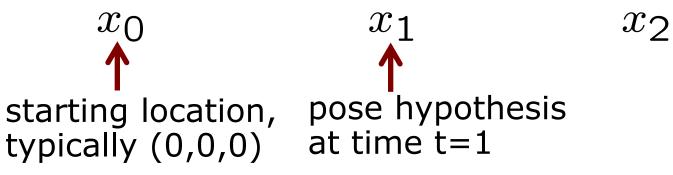
 $p(x_{0:t}, m_{1:M} \mid z_{1:t}, u_{1:t}) =$ $p(x_{0:t} \mid z_{1:t}, u_{1:t}) p(m_{1:M} \mid x_{0:t}, z_{1:t})$ $p(x_{0:t} | z_{1:t}, u_{1:t}) \prod p(m_i | x_{0:t}, z_{1:t})$ 2-dimensional EKFs!

Factorization of the SLAM posterior



Modeling the Robot's Path

- Sample-based representation for p(x_{0:t} | z_{1:t}, u_{1:t})
- Each sample is a path hypothesis



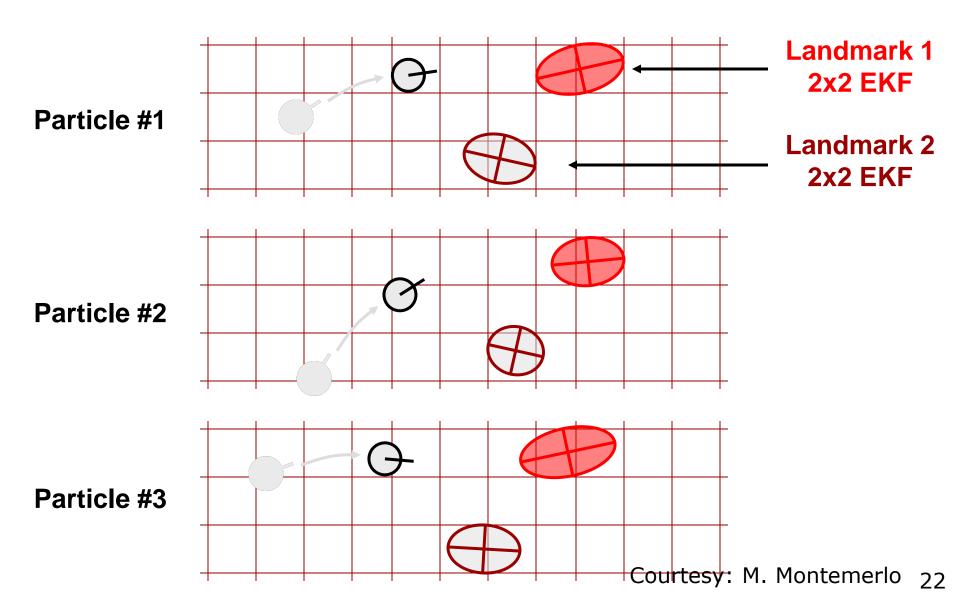
- Past poses of a sample are not revised
- No need to maintain past poses in the sample set

FastSLAM

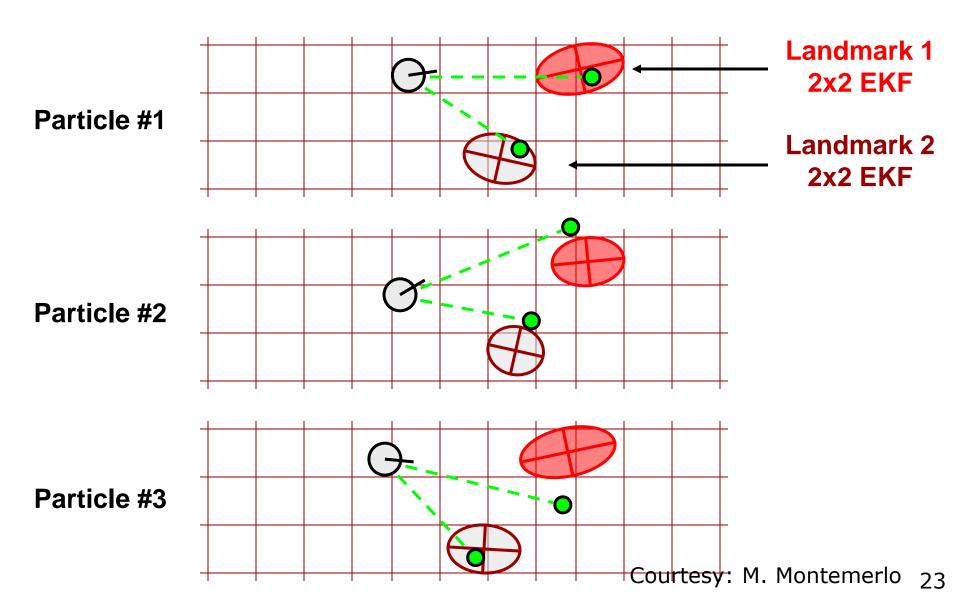
- Proposed by Montemerlo et al. in 2002
- Each landmark is represented by a 2x2 EKF
- Each particle therefore has to maintain M individual EKFs



FastSLAM – Action Update



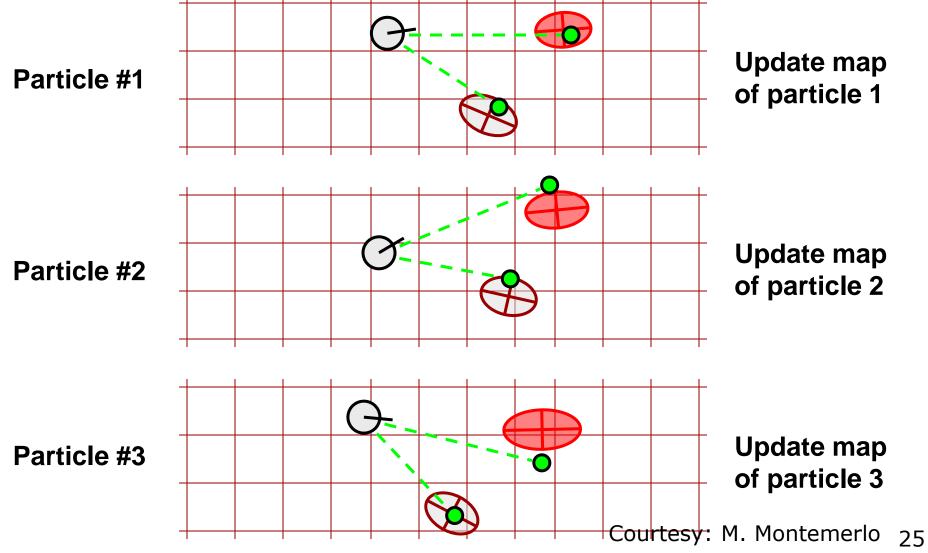
FastSLAM – Sensor Update



FastSLAM – Sensor Update Particle #1 **Weight = 0.8** Weight = 0.4Particle #2 Weight = 0.1**Particle #3**

Courtesy: M. Montemerlo 24

FastSLAM – Sensor Update



Key Steps of FastSLAM 1.0

 Extend the path posterior by sampling a new pose for each sample

 $x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$

- Compute particle weight $w^{[k]} = |2\pi Q|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T Q^{-1} (z_t - \hat{z}^{[k]})\right\}$ measurement covariance
- Update belief of observed landmarks (EKF update rule)
- Resample

FastSLAM 1.0 – Part 1

1: FastSLAM1.0_known_correspondence($z_t, c_t, u_t, \mathcal{X}_{t-1}$):

2: for
$$k = 1$$
 to N do
3: Let $\left\langle x_{t-1}^{[k]}, \left\langle \mu_{1,t-1}^{[k]}, \Sigma_{1,t-1}^{[k]} \right\rangle, \ldots \right\rangle$ be particle k in \mathcal{X}_{t-1}
4: $x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$ // sample pose

FastSLAM 1.0 – Part 1

- 1: FastSLAM1.0_known_correspondence($z_t, c_t, u_t, \mathcal{X}_{t-1}$):
- 2: for k = 1 to N do // loop over all particles Let $\left\langle x_{t-1}^{[k]}, \left\langle \mu_{1,t-1}^{[k]}, \Sigma_{1,t-1}^{[k]} \right\rangle, \ldots \right\rangle$ be particle k in \mathcal{X}_{t-1} 3: $x_{t}^{[k]} \sim p(x_{t} \mid x_{t}^{[k]}, u_{t})$ 4: // sample pose 5: // observed feature $i = c_t$ if feature j never seen before 6: $\mu_{j,t}^{[k]} = h^{-1}(z_t, x_t^{[k]})$ // initialize mean 7: $H = h'(\mu_{i,t}^{[k]}, x_t^{[k]})$ // calculate Jacobian 8: $\Sigma_{j,t}^{[k]} = H^{-1} Q_t (H^{-1})^T \qquad // \text{ initialize covariance} \\ w^{[k]} = p_0 \qquad // \text{ default importance}$ 9: // default importance weight 10: 11: else

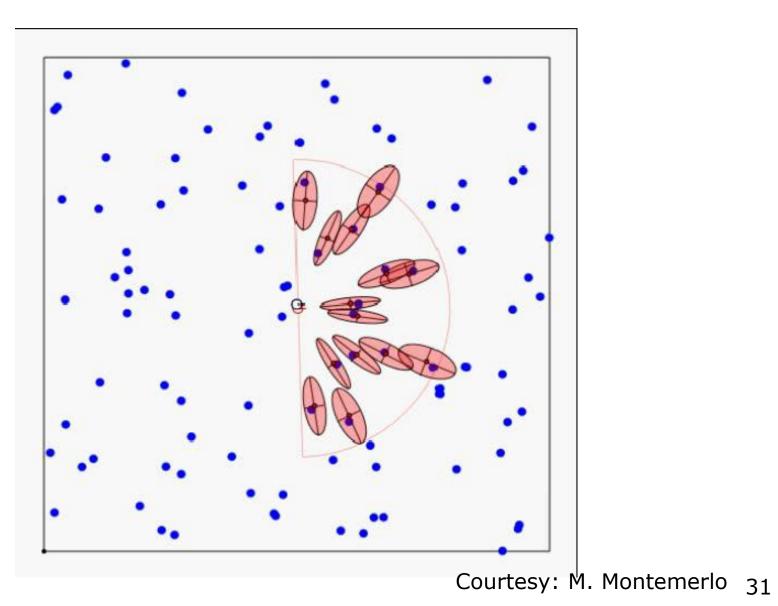
FastSLAM 1.0 – Part 2

11: else
12:
$$\langle \mu_{j,t}^{[k]}, \Sigma_{j,t}^{[k]} \rangle = EKF$$
- $Update(\dots)$ // $update$ landmark
13: $w^{[k]} = |2\pi Q|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T Q^{-1}(z_t - \hat{z}^{[k]})\right\}$
measurement cov. $Q = H \Sigma_{j,t-1}^{[k]} H^T + Q_t$ **exp. observation**
14: endif
15: for all unobserved features j' do
16: $\langle \mu_{j',t}^{[k]}, \Sigma_{j',t}^{[k]} \rangle = \langle \mu_{j',t-1}^{[k]}, \Sigma_{j',t-1}^{[k]} \rangle$ // leave unchanged
17: endfor
18: endfor
19: $\mathcal{X}_t = \text{resample}\left(\left\langle x_t^{[k]}, \left\langle \mu_{1,t}^{[k]}, \Sigma_{1,t}^{[k]} \right\rangle, \dots, w^{[k]} \right\rangle_{k=1,\dots,N}\right)$

FastSLAM 1.0 – Part 2 (long)

11: else		
12:	$ \hat{z}^{[k]} = h(\mu_{j,t-1}^{[k]}, x_t^{[k]}) H = h'(\mu_{j,t-1}^{[k]}, x_t^{[k]}) Q = H \Sigma_{j,t-1}^{[k]} H^T + Q_t K = \Sigma^{[k]} H^T Q^{-1} $	// measurement prediction
13: EVE	$H = h'(\mu_{j,t-1}^{[k]}, x_t^{[k]})$	// calculate Jacobian
	$Q = H \sum_{j,t-1}^{[k]} H^T + Q_t$	// measurement covariance
15:	$K = \Sigma_{j,t-1}^{[k]} H^T Q^{-1}$	// calculate Kalman gain
16:	$ \begin{array}{c} \mathcal{Q} & \Pi \ \mathcal{L}_{j,t-1} \Pi & + \mathcal{Q}_{t} \\ K = \Sigma_{j,t-1}^{[k]} H^{T} \ Q^{-1} \\ \mu_{j,t}^{[k]} = \mu_{j,t-1}^{[k]} + K(z_{t} - \hat{z}^{[k]}) \end{array} $	// update mean
17:	$\Sigma_{j,t}^{[k]} = (I - K H) \Sigma_{j,t-1}^{[k]}$	// update covariance
18:	$w^{[k]} = 2\pi Q ^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})\right\}$	$\left[\right]^{T}$
	$Q^{-1}(z_t - \hat{z}^{[k]})$	// importance factor
19: endif		
20: for all unobserved features j' do		
21:	$\langle \mu_{j',t}^{[k]}, \Sigma_{j',t}^{[k]} angle = \langle \mu_{j',t-1}^{[k]}, \Sigma_{j',t-1}^{[k]} angle$	// leave unchanged
23: endfor		
24: endfor		
25: $\mathcal{X}_t =$	= resample $\left(\left\langle x_t^{[k]}, \left\langle \mu_{1,t}^{[k]}, \Sigma_{1,t}^{[k]} \right\rangle, \dots, \right)$	$\left. w^{[k]} \right\rangle_{k=1,\ldots,N} \right)$
26: retu	$\operatorname{rn} \mathcal{X}_t$,

FastSLAM in Action



The Weight is a Result From the Importance Sampling Principle

- Importance weight is given by the ratio of target and proposal in $\boldsymbol{x}^{[k]}$
- See: importance sampling principle

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

The target distribution is

 $p(x_{1:t} \mid z_{1:t}, u_{1:t})$

The proposal distribution is

$$p(x_{1:t} \mid z_{1:t-1}, u_{1:t})$$

Proposal is used step-by-step

$$p(x_{1:t} \mid z_{1:t-1}, u_{1:t}) = \underbrace{p(x_t \mid x_{t-1}, u_t)}_{\text{from } \mathcal{X}_{t-1} \text{ to } \bar{\mathcal{X}}_t} \underbrace{p(x_{1:t-1} \mid z_{1:t-1}, u_{1:t-1})}_{\mathcal{X}_{t-1}}$$
33

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$

$$= \frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t})}{p(x_{t}^{[k]} \mid x_{t-1}, u_{t}) \ p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$$
$$= \frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t})}{p(x_t^{[k]} \mid x_{t-1}, u_t) p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

Bayes rule + factorization

 $w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$ $p(x_{1:t}^{\lfloor k \rfloor} \mid z_{1:t}, u_{1:t})$ $p(x_t^{[k]} \mid x_{t-1}, u_t) p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})$ $\eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}) p(x_t^{[k]} \mid x_{t-1}^{[k]}, u_t)$ $p(x_{t}^{[k]} \mid x_{t-1}^{[k]}, u_{t})$ $p(x_{1:t-1}^{\lfloor k \rfloor} \mid z_{1:t-1}, u_{1:t-1})$ $p(x_{1:t-1}^{[k]} \mid \overline{z_{1:t-1}, u_{1:t-1}})$

 $w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$ $p(x_{1:t}^{\lfloor k \rfloor} \mid z_{1:t}, u_{1:t})$ $p(x_{t}^{[k]} \mid x_{t-1}, u_{t}) p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})$ $\eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}) p(x_t^{[k]} \mid x_{t-1}^{[k]}, u_t)$ $p(x_{t}^{[k]} \mid x_{t-1}^{[k]}, u_{t})$ $p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})$ $p(x_{1,t-1}^{[k]} \mid z_{1,t-1}, u_{1,t-1})$

 $w^{[k]} = \frac{\operatorname{target}(x^{[k]})}{\operatorname{proposal}(x^{[k]})}$ $p(x_{1:t}^{\lfloor k \rfloor} \mid z_{1:t}, u_{1:t})$ $p(x_{t}^{[k]} \mid x_{t-1}, u_{t}) p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})$ $\eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}) p(x_t^{[k]} \mid x_{t-1}^{[k]}, u_t)$ $p(x_{t}^{[k]} \mid x_{t-1}^{[k]}, u_{t})$ $p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})$ $p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})$ $= \eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1})$

 Integrating over the pose of the observed landmark leads to

$$w^{[k]} =$$

= $\eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1})$
= $\eta \int p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}, m_j) p(m_j \mid x_{1:t}^{[k]}, z_{1:t-1}) dm_j$

 Integrating over the pose of the observed landmark leads to

$$w^{[k]} = = \eta \ p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}) = \eta \ \int p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}, m_j) \ p(m_j \mid x_{1:t}^{[k]}, z_{1:t-1}) \ dm_j = \eta \ \int p(z_t \mid x_t^{[k]}, m_j) \ p(m_j \mid x_{1:t-1}^{[k]}, z_{1:t-1}) \ dm_j$$

 Integrating over the pose of the observed landmark leads to

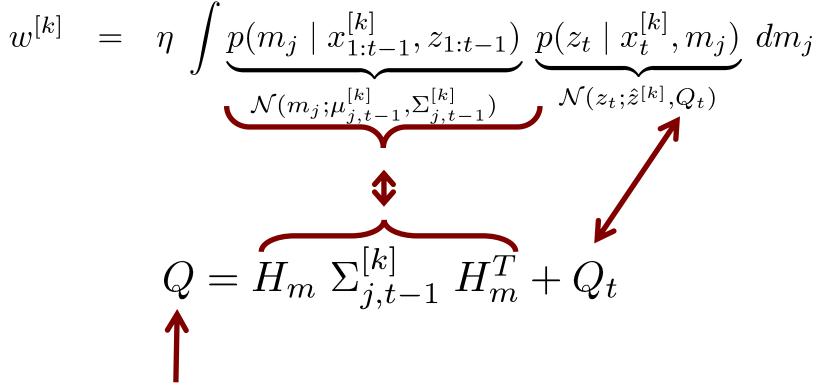
$$w^{[k]} =$$

$$= \eta p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1})$$

$$= \eta \int p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}, m_j) p(m_j \mid x_{1:t}^{[k]}, z_{1:t-1}) dm_j$$

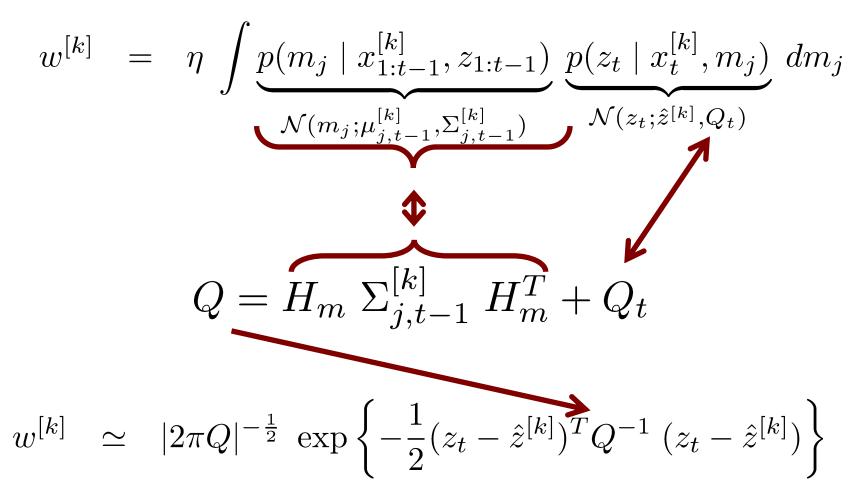
$$= \eta \int \underbrace{p(z_t \mid x_t^{[k]}, m_j)}_{\mathcal{N}(z_t; \hat{z}^{[k]}, Q_t)} \underbrace{p(m_j \mid x_{1:t-1}^{[k]}, z_{1:t-1})}_{\mathcal{N}(m_j; \mu_{j,t-1}^{[k]}, \Sigma_{j,t-1}^{[k]})} dm_j$$

This leads to



measurement covariance (pose uncertainty of the landmark estimate plus measurement noise)

This leads to



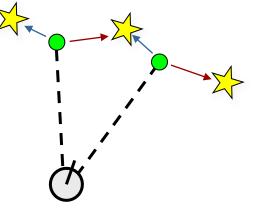
FastSLAM 1.0 – Part 2

11: else
12:
$$\langle \mu_{j,t}^{[k]}, \Sigma_{j,t}^{[k]} \rangle = EKF\text{-}Update(\dots) // update landmark$$

13: $w^{[k]} = |2\pi Q|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T Q^{-1}(z_t - \hat{z}^{[k]})\right\}$
14: endif
15: for all unobserved features j' do
16: $\langle \mu_{j',t}^{[k]}, \Sigma_{j',t}^{[k]} \rangle = \langle \mu_{j',t-1}^{[k]}, \Sigma_{j',t-1}^{[k]} \rangle // \text{ leave unchanged}$
17: endfor
18: endfor
19: $\mathcal{X}_t = \text{resample}\left(\left\langle x_t^{[k]}, \left\langle \mu_{1,t}^{[k]}, \Sigma_{1,t}^{[k]} \right\rangle, \dots, w^{[k]} \right\rangle_{k=1,\dots,N}\right)$
20: return \mathcal{X}_t

Data Association Problem

Which observation belongs to which landmark?

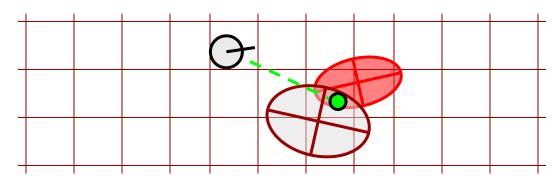


- More than one possible association
- Potential data associations depend on the pose of the robot

Particles Support for Multi-Hypotheses Data Association

- Decisions on a perparticle basis
- Robot pose error is factored out of data association decisions

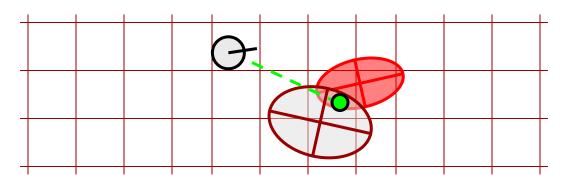
Per-Particle Data Association



Was the observation generated by the **red** or by the **brown** landmark?

P(observation|red) = 0.3 P(observation|brown) = 0.7

Per-Particle Data Association

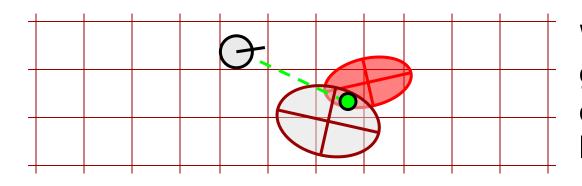


Was the observation generated by the **red** or by the **brown** landmark?

P(observation|red) = 0.3 P(observation|brown) = 0.7

- Two options for per-particle data association
 - Pick the most probable match
 - Pick an random association weighted by the observation likelihoods
- If the probability for an assignment is too low, generate a new landmark

Per-Particle Data Association

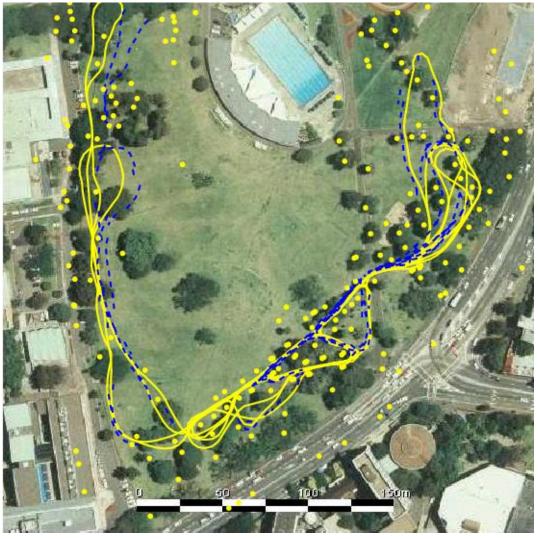


Was the observation generated by the **red** or by the **brown** landmark?

- Multi-modal belief
- Pose error is factored out of data association decisions
- Simple but effective data association
- Big advantage of FastSLAM over EKF

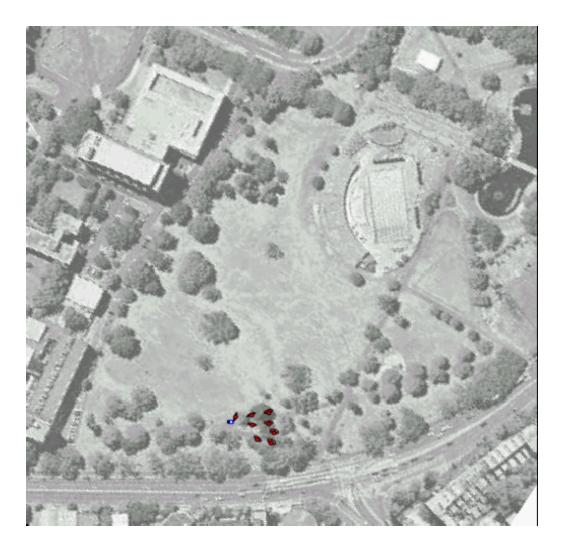
Results – Victoria Park

- 4 km traverse
- < 2.5 m RMS position error
- 100 particles



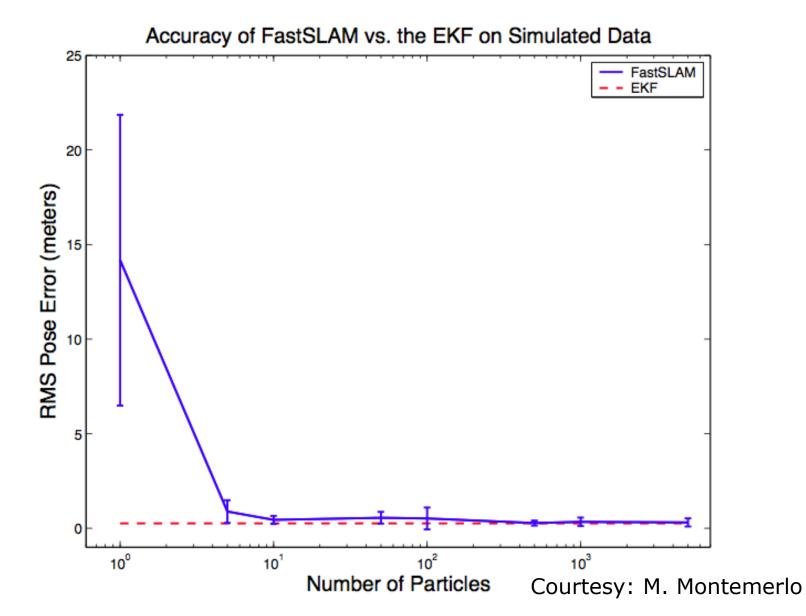
Courtesy: M. Montemerlo

Results – Victoria Park (Video)

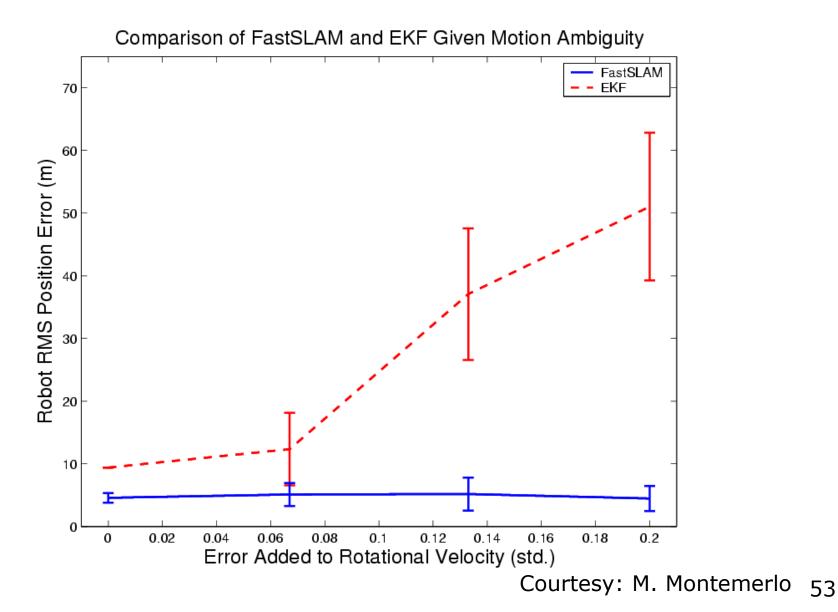


Courtesy: M. Montemerlo

Results (Sample Size)



Results (Motion Uncertainty)



FastSLAM 1.0 Summary

- Use a particle filter to model the belief
- Factors the SLAM posterior into lowdimensional estimation problems
- Model only the robot's path by sampling
- Compute the landmarks given the path
- Per-particle data association
- No robot pose uncertainty in the perparticle data association

FastSLAM Complexity – Simple Implementation

 Update robot particles based on the control

 $\mathcal{O}(N)$

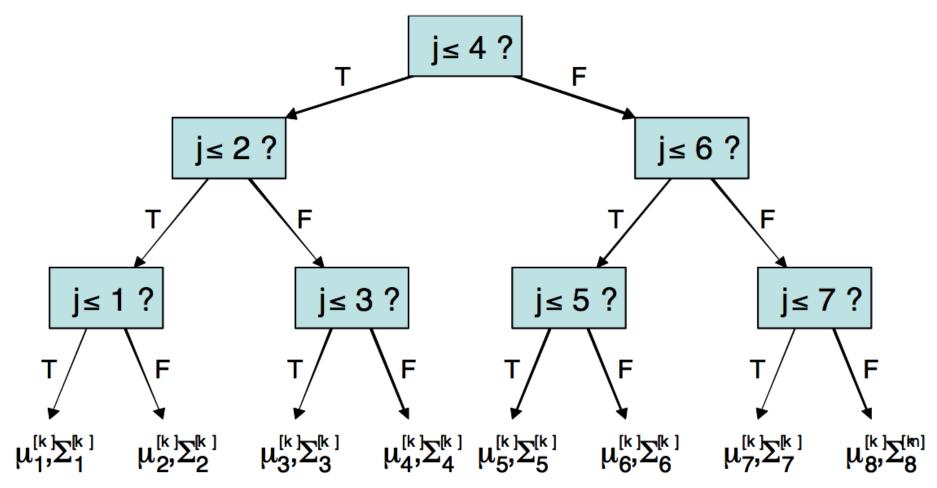
- Incorporate an observation $\mathcal{O}(N)$ into the Kalman filters
- Resample particle set

N = Number of particles M = Number of map features

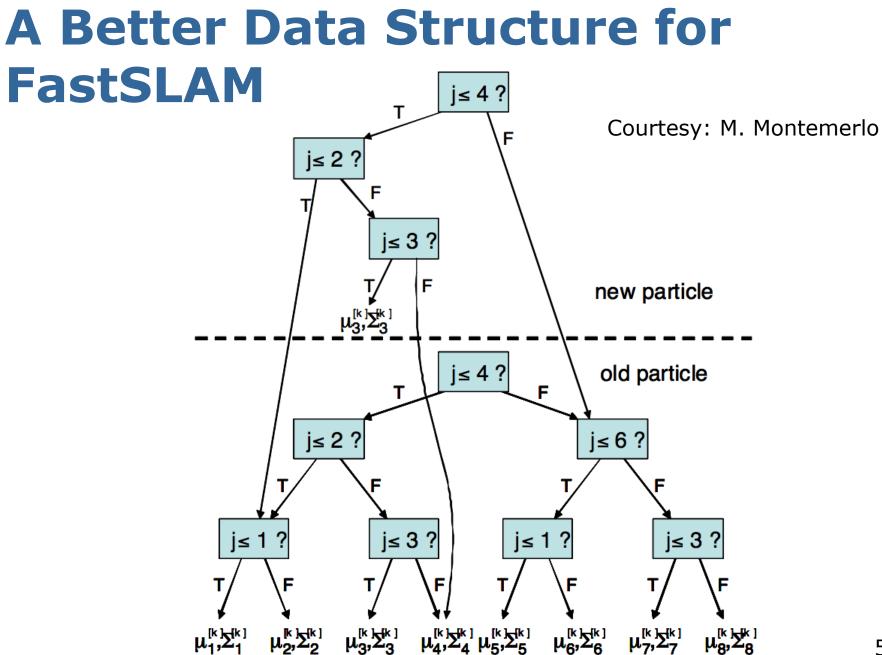
$$\mathcal{O}(NM)$$

 $\mathcal{O}(NM)$

A Better Data Structure for FastSLAM



Courtesy: M. Montemerlo 56



FastSLAM Complexity

 Update robot particles based on the control $\mathcal{O}(N)$

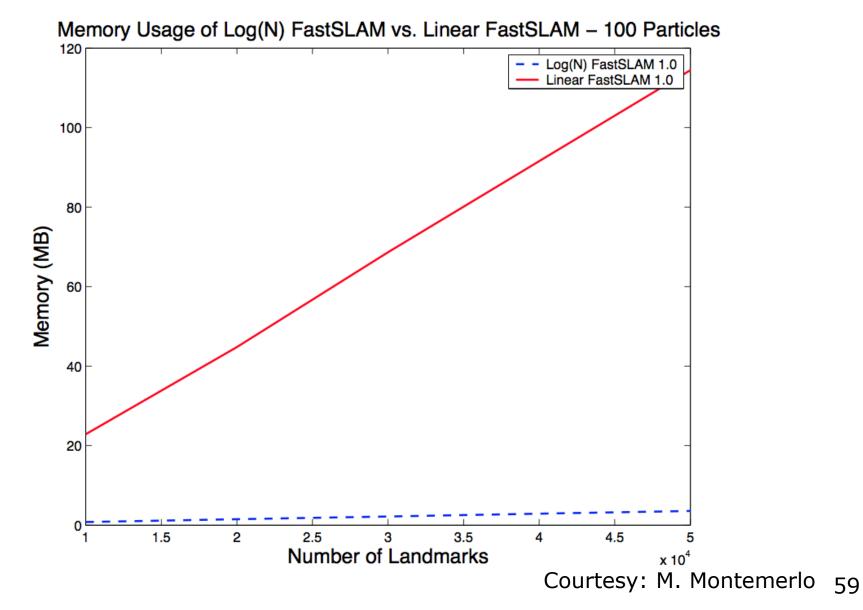
- Incorporate an observation $\ \mathcal{O}(N\log M)$ into the Kalman filters
- Resample particle set

 $\mathcal{O}(N \log M)$

N = Number of particles M = Number of map features



Memory Complexity



FastSLAM 1.0

 FastSLAM 1.0 uses the motion model as the proposal distribution

$$x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$$

 Is there a better distribution to sample from?

FastSLAM 1.0 to FastSLAM 2.0

 FastSLAM 1.0 uses the motion model as the proposal distribution

 $x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$

- FastSLAM 2.0 considers also the measurements during sampling
- Especially useful if an accurate sensor is used (compared to the motion noise)

FastSLAM 2.0 (Informally)

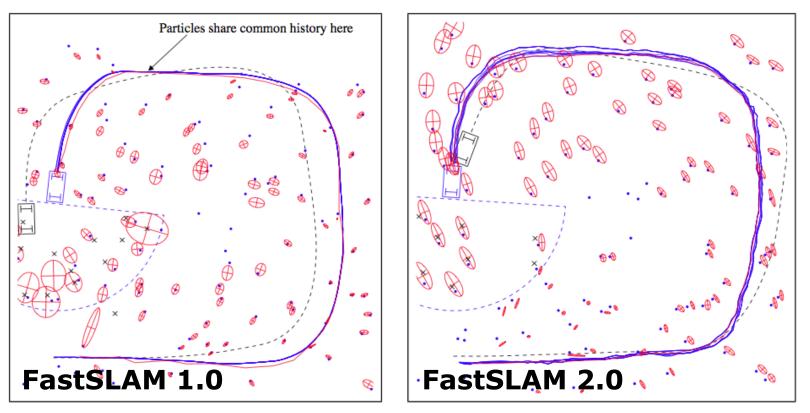
FastSLAM 2.0 samples from

$$x_t^{[k]} \sim p(x_t \mid x_{1:t-1}^{[k]}, u_{1:t}, \underline{z_{1:t}})$$

- Results in a more peaked proposal distribution
- Less particles are required
- More robust and accurate
- But more complex...

FastSLAM Problems

- How to determine the sample size?
- Particle deprivation, especially when closing (multiple) loops



FastSLAM Summary

- Particle filter-based SLAM
- Rao-Blackwellization: model the robot's path by sampling and compute the landmarks given the poses
- Allow for per-particle data association
- FastSLAM 1.0 and 2.0 differ in the proposal distribution
- Complexity $\mathcal{O}(N \log M)$

FastSLAM Results

- Scales well (1 million + features)
- Robust to ambiguities in the data association
- Advantages compared to the classical EKF approach (especially with nonlinearities)

Literature

FastSLAM

- Thrun et al.: "Probabilistic Robotics", Chapter 13.1-13.3 + 13.8 (see errata!)
- Montemerlo, Thrun, Kollar, Wegbreit: FastSLAM: A Factored Solution to the Simultaneous Localization and Mapping Problem, 2002
- Montemerlo and Thrun: Simultaneous Localization and Mapping with Unknown Data Association Using FastSLAM, 2003

Slide Information

- These slides have been created by Cyrill Stachniss as part of the robot mapping course taught in 2012/13 and 2013/14. I created this set of slides partially extending existing material of Edwin Olson, Pratik Agarwal, and myself.
- I tried to acknowledge all people that contributed image or video material. In case I missed something, please let me know. If you adapt this course material, please make sure you keep the acknowledgements.
- Feel free to use and change the slides. If you use them, I would appreciate an acknowledgement as well. To satisfy my own curiosity, I appreciate a short email notice in case you use the material in your course.
- My video recordings are available through YouTube: http://www.youtube.com/playlist?list=PLgnQpQtFTOGQrZ4O5QzbIHgl3b1JHimN_&feature=g-list

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