Albert-Ludwigs-Universität Freiburg, Institut für Informatik
Prof. Dr. Wolfram Burgard, Dr. Daniel Büscher, Lukas Luft, Shengchao Yan
Lecture: Robot Mapping
Winter term 2019/2020

Sheet 3

Topic: Extended Kalman Filter

Due: November 19, 2019

Do not use Octave or other software to solve either of the following exercises.

Exercise 1: Bayes Filter and Extended Kalman Filter

(a) Describe briefly the two main steps of the Bayes filter in your own words.

(b) Describe briefly the meaning of the following probability density functions: $p(x_t \mid u_t, x_{t-1})$, $p(z_t \mid x_t)$, and $\text{bel}(x_t)$, which are processed by the Bayes filter.

(c) Specify the distributions that correspond to the above mentioned three terms in the EKF.

(d) Explain in a few sentences all of the components of the EKF algorithm, i. e., μ_t, Σ_t, $\bar{\mu}_t$, $\bar{\Sigma}_t$, g, G_t^x, R_t, h, H_t^x, Q_t, K_t and why they are needed. Specify the dimensionality of these components.

Exercise 2: Jacobians

(a) Derive the Jacobian matrix G_t^x of the noise-free motion function g with respect to the pose of the robot. Use the odometry motion model as in exercise sheet 1:

$$\begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} = \begin{pmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{pmatrix} + \begin{pmatrix} \delta_{trans}\cos(\theta_{t-1} + \delta_{rot1}) \\ \delta_{trans}\sin(\theta_{t-1} + \delta_{rot1}) \\ \delta_{rot1} + \delta_{rot2} \end{pmatrix}.$$

(b) Derive the Jacobian matrix ${}^{\text{low}}H_t^i$ of the noise-free sensor function h corresponding to the i^{th} landmark:

$$h(\bar{\mu}_t, i) = z_t^i = \begin{pmatrix} r_t^i \\ \phi_t^i \end{pmatrix} = \begin{pmatrix} \sqrt{(\bar{\mu}_{i,x} - \bar{\mu}_{t,x})^2 + (\bar{\mu}_{i,y} - \bar{\mu}_{t,y})^2} \\ \text{atan2}(\bar{\mu}_{i,y} - \bar{\mu}_{t,y}, \bar{\mu}_{i,x} - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{pmatrix},$$

where $(\bar{\mu}_{i,x}, \bar{\mu}_{i,y})^T$ is the position of the landmark, $(\bar{\mu}_{t,x}, \bar{\mu}_{t,y}, \bar{\mu}_{t,\theta})^T$ is the pose of the robot at time t, and r_t^i and ϕ_t^i are the observed range and bearing of the landmark, respectively.

Hint: Use $\frac{\partial}{\partial x}\text{atan2}(y, x) = \frac{-y}{x^2+y^2}$, and $\frac{\partial}{\partial y}\text{atan2}(y, x) = \frac{x}{x^2+y^2}$.