Albert-Ludwigs-Universität Freiburg, Institut für Informatik
Lecture: Robot Mapping by Prof. Dr. Wolfram Burgard
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Winter term 2020/2021

Sheet 3

Topic: Extended Kalman Filter

Due: November 24, 2020

Do not use Octave or other software to solve either of the following exercises.

Exercise 1: Bayes Filter and Extended Kalman Filter

(a) Describe briefly the two main steps of the Bayes filter in your own words.

(b) Describe briefly the meaning of the following probability density functions: $p(x_t \mid u_t, x_{t-1})$, $p(z_t \mid x_t)$, and $\mathrm{bel}(x_t)$, which are processed by the Bayes filter.

(c) Specify the distributions that correspond to the above mentioned three terms in the EKF.

(d) Explain in a few sentences all of the components of the EKF algorithm, i. e., μ_t, Σ_t, $\bar{\mu}_t$, $\bar{\Sigma}_t$, g, G_t^x, R_t, h, H_t^x, Q_t, K_t and why they are needed. Specify the dimensionality of these components.

Exercise 2: Jacobians

(a) Derive the Jacobian matrix G_t^x of the noise-free motion function g with respect to the pose of the robot. Use the odometry motion model as in exercise sheet 1:

$$\begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} = \begin{pmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{pmatrix} + \begin{pmatrix} \delta_{trans} \cos(\theta_{t-1} + \delta_{rot1}) \\ \delta_{trans} \sin(\theta_{t-1} + \delta_{rot1}) \\ \delta_{rot1} + \delta_{rot2} \end{pmatrix}.$$

(b) Derive the Jacobian matrix ${}^{\mathrm{low}}H_t^i$ of the noise-free sensor function h corresponding to the i^{th} landmark:

$$h(\bar{\mu}_t, i) = z_t^i = \begin{pmatrix} r_t^i \\ \phi_t^i \end{pmatrix} = \begin{pmatrix} \sqrt{(\bar{\mu}_{i,x} - \bar{\mu}_{t,x})^2 + (\bar{\mu}_{i,y} - \bar{\mu}_{t,y})^2} \\ \mathrm{atan2}(\bar{\mu}_{i,y} - \bar{\mu}_{t,y}, \bar{\mu}_{i,x} - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{pmatrix},$$

where $(\bar{\mu}_{i,x}, \bar{\mu}_{i,y})^T$ is the position of the landmark, $(\bar{\mu}_{t,x}, \bar{\mu}_{t,y}, \bar{\mu}_{t,\theta})^T$ is the pose of the robot at time t, and r_t^i and ϕ_t^i are the observed range and bearing of the landmark, respectively.

Hint: Use $\frac{\partial}{\partial x}\mathrm{atan2}(y, x) = \frac{-y}{x^2+y^2}$, and $\frac{\partial}{\partial y}\mathrm{atan2}(y, x) = \frac{x}{x^2+y^2}$.