

## Sheet 3

Topic: Extended Kalman Filter

Due: November 17, 20202

Do not use Octave or other software to solve either of the following exercises.

### Exercise 1: Bayes Filter and Extended Kalman Filter

- Describe briefly the two main steps of the Bayes filter in your own words.
- Describe briefly the meaning of the following probability density functions:  $p(x_t | u_t, x_{t-1})$ ,  $p(z_t | x_t)$ , and  $\text{bel}(x_t)$ , which are processed by the Bayes filter.
- Specify the distributions that correspond to the above mentioned three terms in the EKF.
- Explain in a few sentences all of the components of the EKF algorithm, i. e.,  $\mu_t$ ,  $\Sigma_t$ ,  $\bar{\mu}_t$ ,  $\bar{\Sigma}_t$ ,  $g$ ,  $G_t^x$ ,  $R_t$ ,  $h$ ,  $H_t^x$ ,  $Q_t$ ,  $K_t$  and why they are needed. Specify the dimensionality of these components.

### Exercise 2: Jacobians

- Derive the Jacobian matrix  $G_t^x$  of the noise-free motion function  $g$  with respect to the pose of the robot. Use the odometry motion model as in exercise sheet 1:

$$\begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} = \begin{pmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{pmatrix} + \begin{pmatrix} \delta_{trans} \cos(\theta_{t-1} + \delta_{rot1}) \\ \delta_{trans} \sin(\theta_{t-1} + \delta_{rot1}) \\ \delta_{rot1} + \delta_{rot2} \end{pmatrix}.$$

- Derive the Jacobian matrix  ${}^{\text{low}}H_t^i$  of the noise-free sensor function  $h$  corresponding to the  $i^{\text{th}}$  landmark:

$$h(\bar{\mu}_t, i) = z_t^i = \begin{pmatrix} r_t^i \\ \phi_t^i \end{pmatrix} = \begin{pmatrix} \sqrt{(\bar{\mu}_{i,x} - \bar{\mu}_{t,x})^2 + (\bar{\mu}_{i,y} - \bar{\mu}_{t,y})^2} \\ \text{atan2}(\bar{\mu}_{i,y} - \bar{\mu}_{t,y}, \bar{\mu}_{i,x} - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{pmatrix},$$

where  $(\bar{\mu}_{i,x}, \bar{\mu}_{i,y})^T$  is the position of the landmark,  $(\bar{\mu}_{t,x}, \bar{\mu}_{t,y}, \bar{\mu}_{t,\theta})^T$  is the pose of the robot at time  $t$ , and  $r_t^i$  and  $\phi_t^i$  are the observed range and bearing of the landmark, respectively.

*Hint:* Use  $\frac{\partial}{\partial x} \text{atan2}(y, x) = \frac{-y}{x^2+y^2}$ , and  $\frac{\partial}{\partial y} \text{atan2}(y, x) = \frac{x}{x^2+y^2}$ .