Exercise: Implement an EKF SLAM System

Implement an extended Kalman filter SLAM (EKF SLAM) system. To support this task, we provide a small Octave framework (see course website). The framework contains the following folders:

- **data** contains files representing the world definition and sensor readings.
- **octave** contains the EKF SLAM framework with stubs to complete.
- **plots** this folder is used to store images.

The below mentioned tasks should be implemented inside the framework in the directory **octave** by completing the stubs.

After implementing the missing parts, you can run the EKF SLAM system. To do that, change into the directory **octave** and launch Octave. Type `ekf_slam` to start the main loop (this may take some time). The program plots the current belief of the robot (pose and landmarks) in the directory **plots**. Figure 1 depicts some example images of the state of the EKF. You can use the images for debugging and to generate an animation. For example, you can use `ffmpeg` from inside the **plots** directory as follows:

```
ffmpeg -r 10 -i ekf_ %03d. png -b 500000 ekf_slam.mp4
```

(a) Implement the prediction step of the EKF SLAM algorithm in the file `prediction_step.m`. Use the odometry motion model:

\[
\begin{pmatrix}
    x_t \\
    y_t \\
    \theta_t
\end{pmatrix} =
\begin{pmatrix}
    x_{t-1} \\
    y_{t-1} \\
    \theta_{t-1}
\end{pmatrix} +
\begin{pmatrix}
    \delta_{\text{trans}} \cos(\theta_{t-1} + \delta_{\text{rot}1}) \\
    \delta_{\text{trans}} \sin(\theta_{t-1} + \delta_{\text{rot}1}) \\
    \delta_{\text{rot}1} + \delta_{\text{rot}2}
\end{pmatrix}.
\]

Compute its Jacobian $^{low}G_t^{x}$ to construct the full Jacobian matrix $G_t^{x}$:

\[
^{low}G_t^{x} = I +
\begin{pmatrix}
    0 & 0 & -\delta_{\text{trans}} \sin(\theta_{t-1} + \delta_{\text{rot}1}) \\
    0 & 0 & \delta_{\text{trans}} \cos(\theta_{t-1} + \delta_{\text{rot}1}) \\
    0 & 0 & 0
\end{pmatrix}.
\]
For the noise in the motion model assume
\[ R_t = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.01 \end{pmatrix}. \]

(b) Implement the correction step in the file `correction_step.m`. The argument \( z \) of this function is a struct array containing \( m \) landmark observations made at time step \( t \). Each observation \( z(i) \) has an id \( z(i).id \), a range \( z(i).range \), and a bearing \( z(i).bearing \).

Iterate over all measurements \( (i = 1, \ldots, m) \) and compute the Jacobian \( H_t^i \) (see Slide 05 Page 35ff.). You should compute a block Jacobian matrix \( H_t \) by stacking the \( H_t^i \) matrices corresponding to the individual measurements. Use it to compute the Kalman gain and update the system mean and covariance after the for-loop. For the noise in the sensor model assume that \( Q_t \) is a diagonal square matrix as follows
\[ Q_t = \begin{pmatrix} 0.01 & 0 & 0 & \ldots \\ 0 & 0.01 & 0 & \ldots \\ 0 & 0 & 0.01 & \ldots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \in \mathbb{R}^{2m \times 2m}. \]

Some implementation tips:

- While debugging, run the filter only for a few steps by replacing the for-loop in `ekf_slam.m` by something along the lines of `for t = 1:50`.

- The command `repmat` allows you to replicate a given matrix in many different ways and is magnitudes faster than using for-loops.

- When converting implementations containing for-loops into a vectorized form it often helps to draw the dimensions of the data involved on a sheet of paper.

- Many of the functions in Octave can handle matrices and compute values along the rows or columns of a matrix. Some useful functions that support this are `sum`, `sqrt`, and many others.