Exercise: The Unscented Transform

Implement the Unscented Transform in Octave. The implementation should consist of two parts: computing the sigma points and recovering the transformed Gaussian.

(a) Implement the function in `compute_sigma_points.m`, which samples the $2n+1$ sigma points given the mean vector and covariance matrix. You should also compute the corresponding point weights $w^m_{i}$ and $w^c_{i}$ for $i = 0, \ldots, 2n$.

(b) Implement the function in `recover_gaussian.m` to compute the mean and covariance of the resulting distribution given the transformed sigma points and their weights.

To support this task, we provide a small Octave framework on the course website. The tasks described above should be implemented inside the framework in the directory `octave` by completing the stubs. After implementing the missing parts, you can test your solution by running the main script. The program will produce a plot containing both the original and transformed distributions and save it in the `plots` directory.

The code provides three different functions describing transformations applied to the distribution. Test your implementation on each of them by uncommenting the corresponding parts in `transform.m`.

After completing the exercise, try other transformations by implementing them in `transform.m`. Moreover, you can change the parameters $\alpha$ and $\kappa$ in `main.m` for computing $\lambda$ and inspect how this affects the sampled sigma points.

Hint: To compute the square root of the covariance matrix in Octave, you can use the function `sqrtm`. Alternatively, you can compute the Cholesky decomposition using `chol`. 